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PART II
"OPTIMAL BEHAVIOR IN THE PRESENCE
OF UNCONVENTIONAL GAINS FROM TRADE"

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"OPTIMAL BEHAVIOR IN THE PRESENCE OF UNCONVENTIONAL GAINS FROM TRADE"

by

Michael Marrese and Jan Vanous

Abstract

Unconventional gains from trade are defined as the non-market benefits of bilateral agreements which are secured through bilateral preferential trade treatment. Unconventional gains from trade arise because of conflicts among different groups in society, which motivate government officials to choose bilateral preferential trade treatment as a hidden way of generating non-market benefits. Creating bilateral terms of trade which are different from world market terms of trade has been an observable example of bilateral preferential trade treatment.

The customs union literature addresses the issue of preferential trade treatment, but focuses on welfare measured in terms of consumption of traded goods and services. Our analysis focuses on commodities which under particular social traditions and during a particular time period are not traded openly. Moreover, our analysis is applicable in either an auctioneer-type situation with many buyers and sellers or in a bilateral bargaining situation.

The purpose of this paper is to analyze the production, trade and consumption behavior of a country facing bilateral terms of trade which differ from world market terms of trade because of the presence of non-market benefits. Section 1 presents two examples in which social conflict causes the bilateral distortion of world market terms of trade. Both examples utilize three hypothetical countries: POLECON, representing a politicized economy interested in obtaining non-market benefits from EXECON; EXECON, which is a potential external source of non-market benefits for either POLECON or ROW; ROW, the rest of the world, which is also interested in obtaining non-market benefits from EXECON.

In the first example, EXECON's non-market benefits are auctioned to the highest bidder. Suppose POLECON is the highest bidder. Why doesn't POLECON send a lump-sum payment to EXECON for these on-market benefits?

Assume that a conflict exists within POLECON. Suppose POLECON's government values highly the non-market benefits from EXECON but POLECON's population does not. POLECON's government may decide to use a hidden means to purchase the non-market benefits in order to circumvent domestic conflict.

Why might POLECON decide to alter her bilateral terms of trade with EXECON? Anyone not closely associated with POLECON's government would have difficulty in determining world market prices because of the proliferation of grades and qualities of commodities and because of the paucity of detailed trade data. So POLECON could covertly pay for non-market benefits by exporting goods to EXECON at discount prices and importing goods from EXECON at premium prices. Moreover, as long as POLECON either knows, or is able to predict reasonably well, the world market prices of those commodities to which discounts or premiums are to be added, POLECON will be able to calculate the "implicit subsidy" that EXECON is receiving.
The term "implicit subsidy" refers to the payment EXECON receives from POLECON for non-market benefits. It is calculated as the value of exports less imports evaluated at world market prices minus the value evaluated at the preferential bilateral prices (assuming a standard unit of account is used through the world). Therefore, the implicit subsidy is the opportunity loss which POLECON bears by trading with EXECON at preferential bilateral prices rather than with ROW at world market prices.

This example is consistent with the analysis in section 2 as long as the increments in the bids of POLECON and ROW for an additional unit of non-market benefits increases as the quantity of non-market benefits increases.

In the second example, hegemony, defined as a situation in which one country has power superior to that of other countries, plays a crucial role. Assume POLECON has hegemony over EXECON. In the bilateral bargaining situation between POLECON and EXECON over non-market benefits, hegemony confers on POLECON three powers: (1) elimination of ROW as an alternative purchaser of non-market benefits received; (2) control over the amount paid to EXECON for the non-market benefits.

The first element of hegemony restricts EXECON from auctioning non-market benefits to the highest bidder. The second allows POLECON to choose q*, the quantity of non-market benefits to be transferred. The third refers to the set of feasible payments for q*. POLECON will not pay more than the amount of \( v_p(q^*) \) to EXECON for q*, and EXECON will not accept less than the amount \( v_E(q^*) \) from POLECON for q*. So the bilateral bargaining problem is to choose \( a(q^*) \), the actual payment for q*, such that \( v_E(q^*) \leq a(q^*) \leq v_p(q^*) \).

POLECON may not have the power to set \( a(q^*) = v_E(q^*) \), possibly because EXECON will argue that POLECON already has imposed restrictions on EXECON by eliminating the auction process and by choosing q*. Under a plausible set of assumptions (which includes the presence of conflict between POLECON's government and EXECON's population), we argue that as the quantity of non-market benefits increases, POLECON's incremental subsidy to EXECON for an additional unit of non-market benefits increases. In other words, there are diminishing returns to an extra unit of subsidy.

The analysis in section 2 takes place from POLECON's point of view. With a fixed endowment of inputs, POLECON produces goods A, B and C. Only A and B are traded internationally, with POLECON importing A and exporting B. In deciding on the optimal pattern of trade with ROW or EXECON or both, POLECON weighs the relative importance of two considerations. On one hand, the terms of trade offered by ROW are more favorable to POLECON than those offered by EXECON. On the other hand, non-market benefits, obtainable by exchanging B for A with EXECON at relatively disadvantageous terms of trade, act as a substitute for POLECON's domestic production of C.

Two models are presented which incorporate this tradeoff: maximization of the utility derived from both conventional and unconventional gains from trade, and maximization of conventional gains from trade subject to a constraint which is influenced by unconventional gains from trade.
The results of Models 1 and 2 indicate that whether POLECON trades with ROW or EXECON or both depends on the extent to which non-market benefits substitute for C and on POLECON's relative preference for C. When making marginal trade decisions POLECON always chooses the greater of world market terms of trade and the marginal terms of trade inclusive of EXECON's non-market benefits. Finally, POLECON's ability to secure unconventional gains from trade increases her utility, and in the case when A, B and C are normal goods, creates optimal patterns of production and consumption which include greater A and B.
OPTIMAL BEHAVIOR IN THE PRESENCE OF UNCONVENTIONAL GAINS FROM TRADE*

by

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Unconventional gains from trade are defined as the non-market benefits of bilateral agreements which are secured through bilateral preferential trade treatment. Unconventional gains from trade arise because of conflicts among different groups in society, which motivate government officials to choose bilateral preferential trade treatment as a hidden way of generating non-market benefits. Creating bilateral terms of trade which are different from world market terms of trade has been an observable example of bilateral preferential treatment.

The customs union literature addresses the issue of preferential trade treatment, but focuses on welfare measured in terms of the consumption of traded goods and services. Our analysis focuses on commodities which under particular social traditions and during a particular time period are not traded publicly. Moreover, a bilateral bargaining model often provides a better framework within which to analyze non-market benefits than does an auctioneer-type model with many buyers and sellers.

Non-market benefits of bilateral agreements may be categorized into four classes: military, political, economic and ideological. The most important type of military benefits are: (i) creation and maintenance of military alliances; (ii) absence of threat (pacification impact); (iii) availability of military bases in strategic locations; (iv) access to military technology, know-how, and training; (v) proxy intervention (availability of modern-day
international mercenaries); (vi) possibility of importing military services in the case of an internal conflict; and (vii) ability to consume another country's defense services (free-rider aspect of defense as a public good). Potential political benefits include voting along alliance lines in international forums, informal foreign government and media support, and the support and friendship of a foreign population. While most economic benefits of bilateral trade agreements are conventional in nature, several may be considered unconventional: increased economic stability, reduced risk of disrupted flow of strategic commodities, and reduced risk of refusal to purchase a country's exports for reasons other than their price-competitiveness. Possible ideological byproducts of bilateral agreements primarily occur in three areas: acceptance of a desired political ideology and its propagation to other countries; achievement of religious unity; and strengthening of ethnic solidarity.

The purpose of this paper is to analyze the production, trade and consumption behavior of two countries facing bilateral terms of trade which differ from world market terms of trade because of the presence of non-market benefits. Section 1 presents two examples in which social conflict causes the bilateral distortion of world market terms of trade. The first example involves a covert auction of non-market benefits, while the second is based on bilateral bargaining. Both examples illustrate how the amount paid for the non-market benefits might be determined. In section 2, the effects of purchasing non-market benefits via trade conducted at prices which are more favorable to the seller and less favorable to the buyer than world market prices, are studied. Two models are presented: maximization of the utility derived from both conventional and unconventional gains from trade; and maximization of conventional gains from trade subject to a constraint which is influenced by unconventional gains from trade.
Section 1: The Presence of Two Sets of Terms of Trade Due to Social Conflict

In the introduction, unconventional gains from trade are said to arise because of conflicts among the preferences of different groups in society. Two abstract examples serve to illustrate why social conflict is the reason for the existence of unconventional gains from trade.

Consider three countries: POLECON, representing a politicized economy interested in obtaining non-market benefits from EXECON; EXECON, which is a potential external source of non-market benefits for either POLECON or ROW; ROW, the rest of the world, which is also interested in obtaining non-market benefits from EXECON.

In the first example, EXECON's non-market benefits are auctioned to the highest bidder. Suppose POLECON is the highest bidder. Why doesn't POLECON send a lump-sum payment to EXECON for these non-market benefits?

Assume that a conflict exists within POLECON. Suppose POLECON's government values highly the non-market benefits from EXECON but POLECON's population does not. POLECON's government may decide to use a hidden means to purchase the non-market benefits in order to circumvent domestic conflict.

Why might POLECON decide to alter her bilateral terms of trade with EXECON? Anyone not closely associated with POLECON's government would have difficulty in determining world market prices because of the proliferation of grades and qualities of commodities and because of the paucity of detailed trade data. So POLECON could covertly pay for non-market benefits by exporting goods to EXECON at discount prices and importing goods from EXECON at premium prices. Moreover, as long as POLECON either knows, or is able to predict reasonably well, the world market prices of those commodities to which discounts or premiums are to be added, POLECON will be able to calculate the "implicit subsidy" that EXECON is receiving.
The term "implicit subsidy" refers to the payment EXECON receives from POLECON for non-market benefits. It is calculated as the value of exports less imports evaluated at world market prices minus the value evaluated at the preferential bilateral prices (assuming a standard unit of account is used throughout the world). Therefore, the implicit subsidy is the opportunity loss which POLECON bears by trading with EXECON at preferential bilateral prices rather than with ROW at world market prices.

This example is consistent with the analysis in section 2 as long as the increments in the bids of POLECON and ROW for an additional unit of non-market benefits increase as the quantity of non-market benefits increases.

In the second example, hegemony, defined as a situation in which one country has power superior to that of other countries, plays a crucial role. Assume POLECON has hegemony over EXECON. In the bilateral bargaining situation between POLECON and EXECON over non-market benefits, hegemony confers on POLECON three powers: (1) elimination of ROW as an alternative purchaser of non-market benefits; (2) control over the quantity of non-market benefits received; (3) some control over the amount paid to EXECON for the non-market benefits.

The first element of hegemony restricts EXECON from auctioning her non-market benefits to the highest bidder. The second allows POLECON to choose \( q^* \), the quantity of non-market benefits to be transferred. The third refers to the set of feasible payments for \( q^* \). POLECON will not pay more than the amount \( v_p(q^*) \) to EXECON for \( q^* \), and EXECON will not accept less than the amount \( v_E(q^*) \) from POLECON for \( q^* \). So the bilateral bargaining problem is to choose \( a(q^*) \), the actual payment for \( q^* \), such that \( v_E(q^*) \leq a(q^*) \leq v_p(q^*) \). POLECON may not have power to set \( a(q^*) = v_E(q^*) \), possibly because EXECON will argue that POLECON already has imposed restrictions on EXECON by eliminating the auction process and by choosing \( q^* \). However, POLECON's control over price can be quantified, at least
conceptually, by an index of price control $\theta(q^*)$, which is calculated as:

$$\theta(q^*) = 1 - \frac{a(q^*) - v_E(q^*)}{v_p(q^*) - v_E(q^*)}.$$ 

If $\theta(q^*) = 0$, then POLECON has no price control; if $\theta(q^*) = 1$, then POLECON has complete price control.

In the case in which the transfer of non-market benefits from EXECON to POLECON is advantageous or at least non-detrimental to all parties in both countries, then the solution to the bilateral bargaining problem may well be a zero payment to EXECON because EXECON does not have a viable threat position [this is certainly so if $\theta(q^*) = 1$]. However the modelling in section 2 requires that POLECON behaves as if there exists a functional relationship indicating that for an additional unit of non-market benefits, the incremental implicit subsidy increases as the quantity of non-market benefits increases.

The following assumptions provide a framework in which this functional relationship seems plausible: POLECON realizes that EXECON's population, as distinct from EXECON's government, interprets greater military and political association with POLECON as meaning less sovereignty; sovereignty is a commodity desired by EXECON's population. Even if the governments of POLECON and EXECON have the same preferences, as long as EXECON's population has a threat position that forces POLECON to pay a greater additional increment of subsidy for each additional increment of benefits, the analysis in section 2 is valid. So it is the conflict between POLECON's government and EXECON's population which causes the distortion of world market terms of trade.
Section 2: Optimal Behavior

The analysis in this section takes place from POLECON's point of view. POLECON has the option of trading with ROW or EXECON or both. ROW offers POLECON better terms of trade than EXECON (see Table 1, part 6), but we will demonstrate that it may be optimal to trade with both countries simultaneously.

With a fixed endowment of inputs, POLECON produces goods A, B and C. Only A and B are traded internationally, with POLECON importing A and exporting B at existing relative prices. However, non-market benefits obtainable by exchanging B for A with EXECON at relatively disadvantageous terms of trade, act as a substitute for POLECON's production of C.

E(•) (see Table 1, part 10) is a strictly concave and increasing function which maps the implicit subsidy, π, into non-market benefits, E. Thus EXECON is modelled as providing an additional increment of non-market benefits for an increment of implicit subsidy which increases as the quantity of non-market benefits increases.

The implicit subsidy, π, is the opportunity loss which POLECON bears for trading with EXECON (see Table 1, part 8). Given the balance of trade constraints with ROW and EXECON (see Table 1, part 7), π may be reduced to the value of all exports less imports evaluated at world market prices (see Table 1, part 9). So π is equal to POLECON's hypothetical balance of trade surplus that would have existed if POLECON would have conducted all of her trade at world market prices.

D(•) (see Table 1, part 11) captures the substitution between domestically produced C and E. D is the amount of C that yields the same level of services to POLECON as the combination C(π) and E(π). So the concave and increasing function D(•) allows the extent to which non-market benefits act as a
Table 1: Characteristics of POLECON

1. Fixed Endowment of Inputs \((\bar{x}_1, \ldots, \bar{x}_N)\) where \(\bar{x}_i > 0\) for \(i = 1, \ldots, N\)

Inputs may be used domestically to produce A, B, or C. Inputs may not be traded.

For \(i = 1, \ldots, N\)

- \(x_{iA}\) amount of \(i^{th}\) input used to produce A
- \(x_{iB}\) amount of \(i^{th}\) input used to produce B
- \(x_{iC}\) amount of \(i^{th}\) input used to produce C

\[
\begin{align*}
\bar{x}_i &\geq x_{iA} \geq 0 \\
\bar{x}_i &\geq x_{iB} \geq 0 \\
\bar{x}_i &\geq x_{iC} \geq 0
\end{align*}
\]  

Input feasibility constraints

2. Production functions \(A(\cdot), B(\cdot)\) and \(C(\cdot)\) are twice differentiable, strictly concave and increasing functions of inputs. Subscripts denote partial derivatives.

\[
\begin{align*}
A &= A(x_{iA}, \ldots, x_{iA}) \equiv A(\bar{x}_A) \text{ where } A(0) = 0 \text{ and } A_j(0) = \infty \text{ for some } j \\
B &= B(x_{iB}, \ldots, x_{iB}) \equiv B(\bar{x}_B) \text{ where } B(0) = 0 \text{ and } B_j(0) = \infty \text{ for some } j \\
C &= C(x_{iC}, \ldots, x_{iC}) \equiv C(\bar{x}_C) \text{ where } C(0) = 0 \text{ and } C_j(0) = \infty \text{ for some } j
\end{align*}
\]

3. \(A^0\) amount of A domestically consumed

\(B^0\) amount of B domestically consumed

4. At existing relative prices, POLECON imports A and exports B

- \([A(\bar{x}_A) - A^0]\) amount of A which is imported
- \([B(\bar{x}_B) - B^0]\) amount of B which is exported

5. \(\alpha\) fraction of \(-[A(\bar{x}_A) - A^0]\) imported from ROW

\((1-\alpha)\) fraction of \(-[A(\bar{x}_A) - A^0]\) imported from EXECON

\(\beta\) fraction of \([B(\bar{x}_B) - B^0]\) exported to ROW

\((1-\beta)\) fraction of \([B(\bar{x}_B) - B^0]\) exported to EXECON

6. \(P_A\) trading price for A with ROW

\(P_B\) trading price for B with ROW

\(R_A\) trading price for A with EXECON

\(R_B\) trading price for B with EXECON

\(P_B > R_B > 0\) and \(R_A > P_A > 0\)

\(\bar{A}_A = P_A \alpha + R_A (1-\alpha)\) average trading price for A

\(\bar{B}_B = P_B \beta + R_B (1-\beta)\) average trading price for B
7. POLECON's balance of trade constraints with:

\[ \text{ROW} \quad P_A a [A(x_A) - A^0] + P_B b [B(x_B) - B^0] = 0; \]

\[ \text{EXECON} \quad R_A (1-a) [A(x_A) - A^0] + R_B (1-\beta) [B(x_B) - B^0] = 0 \]

8. \( \pi \) is the implicit subsidy in terms of a common international currency transferred to EXECON by POLECON.

\[ \pi = (P_A - R_A)(1-\alpha)[A(x_A) - A^0] + (P_B - R_B)(1-\beta)[B(x_B) - B^0] \]

9. Given that balance of trade constraints with ROW and EXECON will always be required to hold, \( \pi \) may be simplified as follows:

\[ \pi = (P_A - R_A)(1-\alpha)[A(x_A) - A^0] + (P_B - R_B)(1-\beta)[B(x_B) - B^0] + \\
\{(P_A a [A(x_A) - A^0] + P_B b [B(x_B) - B^0]) + (R_A (1-a) [A(x_A) - A^0] + R_B (1-\beta) [B(x_B) - B^0])\}
\]

\[ = P_A [A(x_A) - A^0] + P_B [B(x_B) - B^0] \]

10. \( E \) is a one-dimensional measure of the non-market benefits which POLECON receives from EXECON. \( E(\cdot) \) is twice differentiable, strictly concave and increasing in \( \pi \).

\[ E = E(\pi) \text{ for } \pi > 0 \text{ where } 0 = E(0) \]

11. \( D \) is the amount of \( C \) that yields the same level of services as the combination \( C(x_C) \) and \( E(\pi) \). \( D(\cdot) \) is twice differentiable, concave and increasing in both arguments.

\[ D = D(C(x_C), E(\pi)) \text{ where } 0 = D(C(\bar{C}), E(0)), C(x_C) = D(C(x_C), E(0)) \]
substitute for domestically produced C to vary with the level of domestically produced C and the level of non-market benefits.

Assume that in the absence of unconventional gains from trade, POLECON would trade with the country which offers the more favorable terms of trade, implying that no quality or transportation differences exist.

Finally, assume that POLECON accepts both sets of terms of trade as fixed (though the analysis also holds for terms of trade represented by non-linear offer curves). This assumption ignores POLECON's role in the determination of $R_A$ and $R_B$, the trading prices for A and B with EXECON. However, the analysis in this section does not focus on the determination of $R_A$ and $R_B$, but on POLECON's behavior once world market prices $P_A$ and $P_B$, and $R_A$ and $R_B$ are known.
Model 1: Maximization of Conventional and Unconventional Gains from Trade

Model 1 assumes that POLECON maximizes the utility derived from conventional and unconventional gains from trade subject to production efficiency, a balance of trade constraint with each trading group and feasibility constraints. We posit a utility function, $U[\cdot]$, which is twice differentiable, strictly quasi-concave and increasing in its arguments:

$$(1) \quad U = U[A, B, D(C(XA), E(\pi))]; U_1[0, \ldots, 0] = 0, U_2[\cdot, 0, 0] = 0, U_3[\cdot, 0] = 0.$$  

Another restriction on the utility function is that $A$, $B$ and $C$ are normal goods. So an increase in POLECON's endowment implies increased consumption of $A$, $B$ and $C$.

The constraint of production efficiency is embodied by defining:

$$x_{ic} = x_{ic} - x_{iA} - x_{iB} \quad i = 1, \ldots, N$$

The balance of trade constraints have been embodied by reducing $\pi$ (see Table 1) from:

$$\pi = (P_A - Q_A)(1-\alpha)[A(\bar{x}_A) - A^0] + (P_B - Q_B)(1-\beta)[B(\bar{x}_B) - B^0]$$

to:

$$\pi = P_A[A(\bar{x}_A) - A^0] + P_B[B(\bar{x}_B) - B^0].$$

Actually, this simplification requires only an overall balance of trade constraint with both groups. However only when both individual balance of trade constraints are included, are we able to solve for unique values of $\alpha$ and $\beta$. The unique values for $\alpha$ and $\beta$ enable us to determine POLECON's import and export patterns.

Let us begin by deriving the unique value for $\alpha$ and $\beta$. From the balance of trade constraint with ROW,

$$P_A\alpha[A(\bar{x}_A) - A^0] + P_B\beta[B(\bar{x}_B) - B^0] = 0 \Rightarrow \alpha = -\frac{P_B [B(\bar{x}_B) - B^0]}{P_A [A(\bar{x}_A) - A^0]} \beta.$$  

Substitute this value of $\alpha$ into a balance of trade constraint with EXECON,
\[ R_A \left\{ 1 + \frac{P_B}{P_A} \frac{[B(\bar{X}_B) - B^o]}{[A(\bar{X}_A) - A^o]} \right\} \beta \left[ A(\bar{X}_A) - A^o \right] + R_B (1-\beta) [B(\bar{X}_B) - B^o] = 0 \]

\[ \Rightarrow \frac{[A(\bar{X}_A) - A^o]}{[B(\bar{X}_B) - B^o]} + \frac{P_B}{P_A} \beta + \frac{R_B}{R_A} (1-\beta) = 0 \]

\[ \Rightarrow \left( \frac{P_B}{P_A} - \frac{R_B}{R_A} \right) \beta = - \frac{[A(\bar{X}_A) - A^o]}{[B(\bar{X}_B) - B^o]} - \frac{R_B}{R_A} \Rightarrow \]

\[ \beta = \frac{\frac{[A(\bar{X}_A) - A^o]}{[B(\bar{X}_B) - B^o]} - \frac{R_B}{R_A}}{\frac{P_B}{P_A} - \frac{R_B}{R_A}} \]

(2)

From \[ \alpha = - \frac{P_B}{P_A} \frac{[B(\bar{X}_B) - B^o]}{[A(\bar{X}_A) - A^o]} \beta \] and (2) \Rightarrow

\[ \alpha = - \frac{P_B}{P_A} \frac{[B(\bar{X}_B) - B^o]}{[A(\bar{X}_A) - A^o]} \left\{ \frac{[A(\bar{X}_A) - A^o]}{[B(\bar{X}_B) - B^o]} - \frac{R_B}{R_A} \right\} \]

\[ \Rightarrow \]

\[ \alpha = \frac{P_B}{P_A} + \frac{P_B}{P_A} \frac{R_B}{R_A} \frac{[B(\bar{X}_B) - B^o]}{[A(\bar{X}_A) - A^o]} \]

(3)

So once POLECON's production and consumption of A and B are determined, POLECON's exports to and imports from ROW and EXECON can be calculated by using (2) and (3) respectively.
The feasibility conditions are non-negative inputs and non-negative trade with ROW and with EXECON. Since the production functions are defined only on the non-negative quadrant, the only feasibility condition is \( 0 < \alpha < 1 \) (if this condition is met, then \( 0 < \beta < 1 \) because of the individual balance of trade constraints). Substituting (3) into \( 0 < \alpha < 1 \) =>

\[
0 < \frac{P_B}{P_A} + \frac{P_B}{P_A} \left\{ \frac{[B(B_B) - B^0]}{[A(A_A) - A^0]} \right\} < 1 \Rightarrow \\
\frac{P_B}{P_A} - \frac{R_B}{R_A} = \frac{P_B}{P_A} - \frac{R_B}{R_A}
\]

(4) \( \frac{R_B}{R_A} \leq - \frac{[A(A_A) - A^0]}{[B(B_B) - B^0]} \leq \frac{P_B}{P_A} \)

Hence, including constraint (4) in the model is equivalent to including the restriction of non-negative trade with ROW and with EXECON. If the right-hand weak inequality in (4) holds as an equality at the optimum, then POLECON trades only with ROW. If the left-hand weak inequality holds as an equality at the optimum, then POLECON trades only with EXECON. If (4) is a strict inequality at the optimum, then POLECON trades with ROW and EXECON.

Thus we define \( L \) by including the utility function and constraint (4) as:

(5) \( L = U[A^0, B^0, D(C(\tilde{X}_C), E(\pi))] \)
Since \( U[\cdot] \) is continuous and the constraint set is compact, there exists a feasible solution which maximizes (5). Furthermore, \( U[\cdot] \) is strictly quasi-concave and the constraint set is convex, thus the maximizer is unique. Finally, if the Kuhn-Tucker conditions hold at a feasible point, then this feasible point maximizes (5). Consequently, we wish to test the Kuhn-Tucker conditions over the set of feasible points. Using subscripts attached to functions to denote the first partial derivatives, the Kuhn-Tucker conditions with respect to \( A^0, B^0, x_{iA}, x_{iB} \) and the constraints are:

\[
\begin{align*}
(6) & \quad U_1[\cdot] - U_3[\cdot] D_2[\cdot] E_1(\pi) P_A + \frac{(-\lambda_1 + \lambda_2)}{B(\bar{x}_B) - B^0} = 0; \\
(7) & \quad U_2[\cdot] - U_3[\cdot] D_2[\cdot] E_1(\pi) P_B + (\lambda_1 - \lambda_2) \frac{A(\bar{x}_A) - A^0}{B(\bar{x}_B) - B^0} = 0; \\
(8) & \quad U_3[\cdot] \left[ -D_1[\cdot] C_1(\bar{x}_A) + D_2[\cdot] E_1(\pi) P_A A_1(\bar{x}_A) \right] + (\lambda_1 - \lambda_2) \frac{A_1(\bar{x}_A)}{B(\bar{x}_B) - B^0} = 0; \\
(9) & \quad U_3[\cdot] \left[ -D_1[\cdot] C_1(\bar{x}_B) + D_2[\cdot] E_1(\pi) P_B B_1(\bar{x}_B) \right] + (-\lambda_1 + \lambda_2) \frac{A(\bar{x}_A) - A^0}{B(\bar{x}_B) - B^0} = 0; \\
(10) & \quad \lambda_1 \left[ - \frac{P_B}{P_A} - \frac{A(\bar{x}_A) - A^0}{B(\bar{x}_B) - B^0} \right] = 0; \\
(11) & \quad \lambda_2 \left[ - \frac{B(\bar{x}_B) - B^0}{B(\bar{x}_B) - B^0} \right] = 0; \\
(12) & \quad \lambda_1 \geq 0; \quad (13) \quad \lambda_2 \geq 0.
\end{align*}
\]
We will divide our analysis of the Kuhn-Tucker conditions into three cases which depend on whether the constraints are binding. Case 1 indicates that POLECON only trades with ROW. This implies that constraint 1 is binding, $\lambda_2 = 0, \pi = 0, E(0) = 0, \alpha = \beta = 1$. The solution to case 1 is equivalent to the solution that POLECON would choose in the absence of unconventional gains from trade. Let us denote the solution to case 1 with the superscript "+".

\[
(6) \quad U_1\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A - U_3\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) D_2\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A = 0
\]

\[
(14) \quad \lambda_1 = \left[ U_1\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) - U_3\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) D_2\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A \right] \left[ B\left(\bar{x}_B^+\right) - B^0 \right]. \quad (7) \text{ and } (10) \Rightarrow
\]

\[
(7) \quad U_2\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) - U_3\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) D_2\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A = 0
\]

\[
(15) \quad \lambda_1 = \left[ U_2\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) - U_3\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) D_2\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A \right] \left[ B\left(\bar{x}_B^+\right) - B^0 \right].
\]

Setting (14) = (15) \Rightarrow $U_1\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) - U_3\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) D_2\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A = U_2\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) - U_3\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) D_2\left(\frac{\lambda_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A$

Thus (16) $\frac{U_2\left[ A^0+ B^0\right] D\left(\bar{x}_C^+\right)}{U_1\left[ A^0+ B^0\right] D\left(\bar{x}_C^+\right)} = \frac{P_B}{P_A}$.

\[
(8) \quad U_3\left[ -D_1\left(\frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0}\right) C_1\left(\bar{x}_C^+\right) + D_2\left(\frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A \right] + \frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0} = 0.
\]

Substituting (14) \Rightarrow $U_3\left[ -D_1\left(\frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0}\right) C_1\left(\bar{x}_C^+\right) + D_2\left(\frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A \right] + U_1\left[ -\frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0} \right] = 0 \Rightarrow (17) \frac{A_1\left(\bar{x}_A^+\right)}{C_1\left(\bar{x}_C^+\right)} = \frac{U_3\left[ -D_1\left(\frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0}\right) - U_1\left[ \frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0} \right] \right]}{U_1\left[ \frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0} \right]}$

\[
(9) \text{ and } (10) \Rightarrow U_3\left[ -D_1\left(\frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0}\right) C_1\left(\bar{x}_C^+\right) + D_2\left(\frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0}\right) E_1(0) P_A \right] + \frac{P_B}{P_A} \frac{\lambda_1 A_1}{B\left(\bar{x}_B^+\right) - B^0} = 0.$
Substituting (15) \( \Rightarrow U_3[\cdot] \left[ -D_1(\cdot)C_1(x^+_C) + D_2(\cdot)E_1(0)P_B B_1(x^+_B) \right] + U_2[\cdot]B_1(x^+_B) - \\
U_3[\cdot]D_2(\cdot)E_1(0)P_B B_1(x^+_B) = 0 \Rightarrow \frac{B_1(x^+_B)}{C_1(x^+_C)} = \frac{U_3[\cdot]D_1(\cdot)}{U_2[\cdot]} \)

(17) and (18) \( \Rightarrow \) (19) \( \frac{A_1(x^+_A)}{B_1(x^+_B)} = \frac{U_2[\cdot]}{U_1[\cdot]} \)

So at the optimum for case 1, (16) indicates that the marginal rate of substitution equals world market prices, whereas (19) says that the marginal rate of transformation in production equals the marginal rate of substitution. Thus the standard results hold for case 1.

We have not yet indicated the conditions under which \( \lambda_1 \geq 0 \). From (14) and (15) we know that \( \lambda_1 \geq 0 \) iff

\[ \frac{U_1[\cdot]}{P_A} \geq U_3[\cdot]D_2(\cdot)E_1(0) \text{ and } \frac{U_2[\cdot]}{P_B} \geq U_3[\cdot]D_2(\cdot)E_1(0). \] 

Equality of these relationships implies that \( \lambda_1 = 0 \), which is the limit of case 2 (trade with both countries) as \( \alpha \) and \( \beta \) approach 1.

\[ \lambda_1 > 0 \iff \frac{U_1[\cdot]}{P_A} > U_3[\cdot]D_2(\cdot)E_1(0) \text{ and } \frac{U_2[\cdot]}{P_B} > U_3[\cdot]D_2(\cdot)E_1(0). \] 

These inequalities indicate that no trade with EXECO takes place because the marginal cost of obtaining non-market benefits is greater than the marginal gain from non-market benefits.

Consequently the unique solution for case 1 (\( \lambda_1 > 0 \)) is denoted by \( S_1 \):

\[ S_1 = \{(A(x^+_A), C(x^+_C), B(x^+_B), \alpha^+, \beta^+, \omega^+): \frac{A_1(x^+_A)}{B_1(x^+_B)} = \frac{P_A}{U_1[\cdot]}, \frac{A_1(x^+_A)}{C_1(x^+_C)} = \frac{U_3[\cdot]D_1(\cdot)}{U_1[\cdot]} \}; \]

\[ \frac{U_1[\cdot]}{P_A} > U_3[\cdot]D_2(\cdot)E_1(0); \quad x^+_iA > 0; \quad x^+_iB \geq 0; \quad x^+_iC \geq 0 \quad i = 1, \ldots, \omega \}. \]
Case 2 refers to the instance in which POLECON trades with ROW and EXECON. We know from case 1 that POLECON does not trade with EXECON because the marginal cost of obtaining additional non-market benefits is greater than the marginal gain from additional non-market benefits. Therefore case 2 occurs iff POLECON receives more non-market benefits from EXECON for each amount of implicit subsidy (a shift in $E(\cdot)$, at least near $\pi = 0$) or there is an increase in the extent to which each quantity of non-market benefits substitutes for $C$ (a shift in $D(\cdot)$). So let us assume that either $E(\cdot)$ or $D(\cdot)$ shifts so that that at $S^1$ the following holds:

$$\frac{U_1[\cdot]}{p_A} < U_3[\cdot]D_2[\cdot]E_1(0) \text{ and } \frac{U_2[\cdot]}{p_B} < U_3[\cdot]D_2[\cdot]E_1(0).$$

For case 2 neither constraint is binding, which implies $\lambda_1 = 0, \lambda_2 = 0, \pi > 0, E(\pi) > 0, 0 < \alpha < 1, 0 < \beta < 1$. Let us denote the solution to case 2 with the superscript "*".

\begin{align*}
\frac{U_1[\cdot]}{p_A} & = \frac{U_2[\cdot]}{p_B} \Rightarrow U_3[\cdot]D_2[\cdot]E_1(\pi^*)P_A.
\end{align*}

\begin{align*}
\frac{U_2[\cdot]D_2[\cdot]E_1(\pi^*)}{U_1[\cdot]} P_B & = \frac{p_B}{p_A} \quad \text{and}
\end{align*}

\begin{align*}
\frac{U_3[\cdot]}{p_A} & = \frac{U_3[\cdot]}{p_B} \Rightarrow U_3[\cdot]D_2[\cdot]E_1(\pi^*)P_A.
\end{align*}

\begin{align*}
\Rightarrow U_3[\cdot] \left[ - D_1[\cdot]C_1(x_C^*) + D_2[\cdot]E_1(\pi^*)P_A \right] = 0 \Rightarrow
\end{align*}

\begin{align*}
A_1(x_A^*) & = \frac{D_1[\cdot]C_1(x_C^*)}{D_2[\cdot]E_1(\pi^*)} \frac{U_3[\cdot]}{U_3[\cdot]} D_1[\cdot]C_1(x_C^*).
\end{align*}
(9) \[ \Rightarrow U_3[\cdot] \left[ -D_1(\cdot)C_i(x^*_C) + D_2(\cdot)E_1(\pi^*)P_B B_1(x^*_B) \right] = 0 \Rightarrow \\

(23) \[ B_1(x^*_B) = \frac{D_1(\cdot)C_i(x^*_C)}{D_2(\cdot)E_1(\pi^*)P_B} = \frac{U_3[\cdot]}{U_2[\cdot]} D_1(\cdot)C_i(x^*_C). \]

(22) and (23) \[ \Rightarrow (24) \frac{A_i(x^*_A)}{B_1(x^*_B)} = \frac{P_B}{P_A}. \]

Consequently the unique solution for case 2 is denoted \( S^2 \):

\[ S^2 = \{[A(x^*_A), A^0, B(x^*_B), B^0, C(x^*_C), E(\pi^*)]: \frac{A_i(x^*_A)}{B_1(x^*_B)} = \frac{P_B}{P_A} = \frac{U_2[\cdot]}{U_1[\cdot]} ; \]

\[ \frac{A_i(x^*_A)}{C_i(x^*_C)} = \frac{U_3[\cdot]}{U_1[\cdot]} ; \frac{U_1[\cdot]}{P_A} = U_3[\cdot]D_2(\cdot)E_1(\pi^*) ; \]

\[ x^*_A > 0; x^*_B > 0; x^*_C > 0 \quad i = 1, \ldots, N. \]

POLECON is better off in case 2 than in case 1 because of the greater value of non-market benefits in case 2 (due to the shift in \( E(\cdot) \) or \( D(\cdot) \)). However according to (24) the ratio of world market prices still equals the marginal rate of transformation in production because POLECON, on the margin, trades with ROW. (22) and (23) yield another standard result: POLECON'S level of trade with EXECON and level of domestic production of C are determined so that the marginal rates of transformation in production between A and C and between B and C equal the respective marginal rates of substitution. (21) indicates that at \( S^2 \), the marginal gain from acquiring non-market benefits equals the marginal cost.

For case 2, as compared to case 1, the minimum resource cost of consuming C has decreased due to the shift in the value of non-market benefits. This is equivalent to a non-linear decrease in the per unit cost of C (or a non-linear price decrease), so more C will be consumed due to the pure substitution effect and due to the income effect (C is a normal good). Also, more A and B are consumed because they are normal goods. Thus,
POLECON must produce more A and B for two reasons: to subsidize EXECON and to increase domestic consumption. Given POLECON's fixed endowment and the greater production of A and B, POLECON must produce less C.

Case 3 refers to the situation in which POLECON trades only with EXECON. Therefore, relative to case 2, 3 occurs if POLECON receives more non-market benefits from EXECON for each amount of implicit subsidy (a shift in $E(\cdot)$, at least near $\pi = \pi^\ast$) or there is an increase in the extent to which each quantity of non-market benefits substitutes for C (a shift in $D(\cdot)$). Furthermore, the shift in either $E(\cdot)$ or $D(\cdot)$ must be great enough so that at the solution for case 3 ($\lambda_2 > 0$), the marginal gain of acquiring additional non-market benefits is greater than the marginal cost. For case 3, constraint 2 is binding, which implies $\lambda_1 = 0, \pi > 0, \alpha = \beta = 0$. Let us denote the solution to case 3 with the superscript "^3".

\[
(6) \Rightarrow U_1[\cdot] - U_3[\cdot]D_2[\cdot] E_1(\pi^\ast)P_A + \frac{\lambda_2}{[B(\tilde{\pi}_B^\ast) - B^{O^\ast}]} = 0 \Rightarrow
\]

\[
(25) \lambda_2 = \{U_3[\cdot]D_2[\cdot]E_1(\pi^\ast)P_A - U_1[\cdot]\}[B(\tilde{\pi}_B^\ast) - B^{O^\ast}].
\]

\[
(7) \Rightarrow U_2[\cdot] - U_3[\cdot]D_2[\cdot]E_1(\pi^\ast)P_B - \lambda_2 \frac{[A(\tilde{\pi}_A^\ast) - A^{O^\ast}]}{[B(\tilde{\pi}_B^\ast) - B^{O^\ast}]^2} = 0 \Rightarrow
\]

\[
(26) \lambda_2 = \{U_3[\cdot]D_2[\cdot]E_1(\pi^\ast)P_B - U_2[\cdot]\} \frac{R_A}{R_B} \frac{B(\tilde{\pi}_B^\ast) - B^{O^\ast}}{[B(\tilde{\pi}_B^\ast) - B^{O^\ast}]} \quad \text{since}
\]

\[
\frac{R_B}{R_A} = \frac{[A(\tilde{\pi}_A^\ast) - A^{O^\ast}]}{[B(\tilde{\pi}_B^\ast) - B^{O^\ast}]} \quad \text{from (11)}.
\]

Setting (25)=(26)\Rightarrow

\[
U_3[\cdot]D_2[\cdot]E_1(\pi^\ast)P_A - U_1[\cdot] = \left[U_3[\cdot]D_2[\cdot]E_1(\pi^\ast)P_B - U_2[\cdot]\right] \frac{R_A}{R_B} \Rightarrow
\]
\[ U_3[\cdot]D_2[\cdot]E_1(\pi') = U_1[\cdot] - U_2[\cdot] \frac{R_A}{R_B} \Rightarrow \]

\[ \frac{P_A - P_B}{R_A \cdot R_B} \]

(27) \[ U_3[A^{o'},B^{o'},D(C(\bar{\pi}_C),E(\pi'))]D_2[\cdot]E_1(\pi') = \frac{U_1[\cdot]R_A - U_2[\cdot]R_A}{P_A \cdot R_B - P_B \cdot R_A} \]

(28) \[ B_1(\bar{\pi}_B) = \frac{U_3[\cdot]}{C_1(\bar{\pi}_C)} \]

(29) \[ B_1(\bar{\pi}_B) = \frac{U_3[\cdot]}{U_2[\cdot]} \]

(28) and (29) \[ \frac{A_1(\bar{\pi}_A)}{E_1(\bar{\pi}_B)} = \frac{U_2[A^{o'},B^{o'},D(C(\bar{\pi}_C),E(\pi'))]}{U_1[A^{o'},B^{o'},D(C(\bar{\pi}_C),E(\pi'))]} \]

We have not yet indicated the conditions under which \( \lambda_2 \geq 0 \). From (25), we know \( \lambda_2 \geq 0 \) iff \( U_3[\cdot]D_2[\cdot]E_1(\pi')P_A \geq U_1[\cdot] \). So at the solution for case 3 when \( \lambda_2 > 0 \), the marginal gain of acquiring additional non-market benefits exceeds marginal cost. Now let us examine the necessary and sufficient conditions for this to hold. To begin, we return to (27).
(27) \[ U_3[\cdot]D_2[\cdot]E_1(\pi^-) = \frac{U_1[\cdot]R_B - U_2[\cdot]R_A}{P_A R_B - P_B R_A} \Rightarrow \]
\[ U_3[\cdot]D_2[\cdot]E_1(\pi^-) = U_1[\cdot]R_B \left[ 1 - \frac{U_2[\cdot]R_A}{U_1[\cdot]R_B} \right] \]
\[ P_A R_B \left[ 1 - \frac{P_B R_A}{P_A R_B} \right] \]

(31) \[ U_3[\cdot]D_2[\cdot]E_1(\pi^-)P_A = U_1[\cdot] \left[ 1 - \frac{U_2[\cdot]R_A}{U_1[\cdot]R_B} \right] \]

Consequently \[ U_3[\cdot]D_2[\cdot]E_1(\pi^-)P_A \geq U_1[\cdot] \leftrightarrow \left[ 1 - \frac{U_2[\cdot]R_A}{U_1[\cdot]R_B} \right] \geq 1. \]
From (25) and (31),

\[ \lambda_2 = 0 \leftrightarrow U_3[\cdot]D_2[\cdot]E_1(\pi^-)P_A = U_1[\cdot] \leftrightarrow \frac{U_2[\cdot]}{U_1[\cdot]} = \frac{P_B}{P_A}. \]

Case 3 with \( \lambda_2 = 0 \) is the limit of case 2 as \( \alpha \) and \( \beta \) approach 0.

Now we must determine when \( \lambda_2 > 0 \). Notice that the assumption

\[ \frac{P_B}{P_A} > \frac{R_B}{R_A} \Rightarrow \frac{P_B R_A}{P_A R_B} > 1 \Rightarrow 0 > 1 - \frac{P_B R_A}{P_A R_B}. \]
Thus if \[1 - \frac{U_2[\cdot]R_A}{U_1[\cdot]R_B} > 1 \Rightarrow 1 - \frac{U_2[\cdot]R_A}{U_1[\cdot]R_B} < 1 - \frac{P_BR_A}{P_AR_B}\]

\[\Rightarrow (32) \quad \frac{U_2[\cdot]}{U_1[\cdot]} > \frac{P_B}{P_A} \iff \lambda_2 > 0.\]

In (32), \(\frac{U_2[\cdot]}{U_1[\cdot]}\) may be interpreted as the marginal terms of trade with EXECON inclusive of the non-market benefits. So POLECON does not trade with ROW because the marginal terms of trade with EXECON inclusive of the non-market benefits are greater than world market terms of trade.

Therefore the unique solution for case 3 with \(\lambda_2 > 0\) is denoted as \(S^3:\)

\[S^3 = \{[A(x^*_{A}), A^0, B(x^*_{B}), B^0, C(x^*_{C}), E(\pi^*)]: \quad \frac{A_1(x^*_{A})}{B_1(x^*_{B})} = \frac{U_2[\cdot]}{U_1[\cdot]} > \frac{P_B}{P_A}; \]

\[\frac{A_1(x^*_{A})}{C_1(x^*_{C})} = \frac{U_1[\cdot]D_1[\cdot]}{U_1[\cdot]} ; \quad U_3[\cdot]D_2[\cdot]E_1(\pi^*) = \frac{U_1[\cdot]R_B - U_2[\cdot]R_A}{P_AR_B - P_BR_A}; \]

\[x^*_{iA} \geq 0; x^*_{iB} \geq 0; x^*_{iC} \geq 0 \quad i = 1, \ldots, N.\]

For case 3 as compared to case 2, the minimum resource cost of consuming C has decreased due to the shift in the value of non-market benefits. This is equivalent to a non-linear decrease in the per unit cost of C (or a non-linear price decrease), so more C will be consumed due to the pure substitution effect and due to the income effect (C is a normal good). Also, more A and B will be consumed because they are normal goods. Thus POLECON must produce more A and B for two reasons: to subsidize EXECON and to increase domestic consumption.

Given POLECON's fixed endowment and the greater production of A and B, POLECON
must produce less C.

Adding these results to those previously discussed, we find that:

\[
\begin{align*}
(33) \quad A_R(\bar{x}_R^-) > A_R(\bar{x}_R^+) > A_R(\bar{x}_R^+) ; & \quad B_R(\bar{x}_B^-) > B_R(\bar{x}_B^+) > B_R(\bar{x}_B^+) ; \\
C_R(\bar{x}_C^-) < C_R(\bar{x}_C^+) < C_R(\bar{x}_C^+) ; & \quad A_0^- > A_0^+ > A_0^+ ; \\
D(R_0^+, E(\pi^-)) > D(R_0^+, E(\pi^+)) > D(R_0^+, E(0)) ; & \quad U[A_0^-, B_0^-, D(\cdot)] > U[A_0^+, B_0^+, D(\cdot)] > U[A_0^+, B_0^+, D(\cdot)].
\end{align*}
\]

Another comparison focuses on the marginal gain from acquiring non-market benefits evaluated at \(S^1, S^2\) and \(S^3\). At \(S^1\), it is less than the marginal cost, whereas at \(S^2\) and \(S^3\) it is equal to the marginal cost. However, while the marginal gain is expressed by the same functional form in cases 1, 2 and 3, the expression for marginal cost is

\[
\frac{U_1(\cdot)}{P_A} = \frac{1}{1 - \frac{P_A}{U_1(\cdot)} \frac{R_A}{R_B}} \left[ \frac{1 - \frac{U_2(\cdot)}{U_1(\cdot)} \frac{R_A}{R_B}}{1 - \frac{P_B}{P_A} \frac{R_A}{R_B}} \right]
\]

for case 3 and \(\frac{U_1(\cdot)}{P_A}\) for cases 1 and 2. The expression for marginal cost in cases 1 and 2 is the normalized utility loss of not consuming A. In case 3, the expression for marginal cost is the normalized utility loss of not consuming A multiplied by a factor which adjusts for the implicit subsidy to EXECON.

Finally, at \(S^3\) the marginal rate of transformation in production equals \(\frac{U_2(\cdot)}{U_1(\cdot)}\) but not \(\frac{P_B}{P_A}\) as at \(S^1\) and \(S^2\).
Model 2: Maximization of Conventional Gains from Trade Given the Existence of Unconventional Gains from Trade

Model 2 assumes that decisionmakers wish to maximize the utility derived from conventional gains from trade subject to maintenance of a target level of \( D \), production efficiency, a balance of trade constraint with each country and feasibility conditions.

Under Model 2, the objective function of decisionmakers, \( V[\cdot] \), is twice differentiable, strictly quasi-concave and increasing in \( A^0 \) and \( B^0 \):

\[
V = V[A^0, B^0].
\]

The maintenance of a target level of \( D, \hat{D} \), is expressed as:

\[
\hat{D} = D[C(\bar{X}_C), E(\pi)].
\]

All other constraints are embodied in the same manner as in Model 1.

Now let us define \( L \) as:

\[
L = V[A^0, B^0] + \lambda_0[D[C(\bar{X}_C), E(\pi)] - \hat{D}]
- \lambda_1 \left\{ \frac{P_B}{P_A} - \frac{[A(\bar{X}_A) - A^0]}{[B(\bar{X}_B) - B^0]} \right\} - \lambda_2 \left\{ \frac{R_B}{R_A} + \frac{[A(\bar{X}_A) - A^0]}{[B(\bar{X}_B) - B^0]} \right\}.
\]

Just as for Model 1, if the Kuhn-Tucker conditions for Model 2 hold at a feasible point, then the feasible point is a unique maximum. The Kuhn-Tucker conditions are:

\[
V_1[\cdot] - \lambda_0 D_2[\cdot] E_1(\pi) P_A + \frac{(-\lambda_1 + \lambda_2)}{[B(\bar{X}_B) - B^0]} = 0;
\]

\[
V_2[\cdot] - \lambda_0 D_2[\cdot] E_1(\pi) P_B + (\lambda_1 - \lambda_2) \frac{[A(\bar{X}_A) - A^0]}{[B(\bar{X}_B) - B^0]^2} = 0;
\]

\[
\lambda_0 [-d_1[\cdot] C_1(\bar{X}_C) + D_2[\cdot] E_1(\pi) P_A A_1(\bar{X}_A)] + (\lambda_1 - \lambda_2) \frac{A_1(\bar{X}_A)}{[B(\bar{X}_B) - B^0]} = 0;
\]
\[ \lambda_0 \left[ -D_1(\cdot)C_i(\tilde{x}_C) + D_2(\cdot)E_1(\pi)P_B^iB_1(\tilde{x}_B) \right] + (-\lambda_1 + \lambda_2) \frac{[A(\tilde{x}_A) - A^0]}{[B(\tilde{x}_B) - B^0]^2} B_1(\tilde{x}_B) = 0; \]

\[ \lambda_0 \left[ D(C(\tilde{x}_C), E(\pi)) - \tilde{B} \right] = 0 \]

\[ \lambda_1 \left[ -\frac{P_B}{P_A} \right] = 0; \]

\[ \lambda_2 \left[ \frac{R_B}{R_A} + \frac{[A(\tilde{x}_A) - A^0]}{[B(\tilde{x}_B) - B^0]} \right] = 0; \]

\[ \lambda_0 \geq 0; \quad \lambda_1 \geq 0; \quad \lambda_2 \geq 0. \]

A comparison of (37)-(40), (42), (43), (45) and (46) with (6)-(13) indicates that the two sets of equations are exactly alike except that \( \lambda_0 \) appears in the Model 2 set of equations instead of \( U_3[\cdot] \). Furthermore, the Kuhn-Tucker conditions for Model 2 contain (41) and (44). \( \lambda_0 \), the shadow price of \( C \), is non-zero by (1), so (41) implies a smaller set of feasible production optima for Model 2 relative to Model 1. This means that the decisions in Model 2 concerning the domestic production of \( C \) are less sensitive to the full range of relative prices. This may be seen in Figures 1 and 2.

Figure 1 depicts the set of feasible production optima for Model 1 as the checkered surface in \((A,B,C)\) space. This set is the same regardless of how POLECON evaluates the non-market benefits available from EXECON.

In order to understand Figure 2, we define the maximum amount of \( A, B \) and \( C \) that can be produced if all inputs are employed to produce one type of output good as follows:

\[ A^M = A(\tilde{x}_1, \ldots, \tilde{x}_N), \quad B^M = B(\tilde{x}_1, \ldots, \tilde{x}_N) \quad \text{and} \quad C^M = C(\tilde{x}_1, \ldots, \tilde{x}_N). \]

Next we denote as \( \hat{A} \) and \( \hat{B} \) the maximum amounts of \( A \) and \( B \) that can be produced, given that \( \hat{B} \) is maintained solely through domestic production of \( C \).
Let us assume that \( \hat{D} < C_M \). Immediately it is evident that the set of 
feasible production optima is reduced by the extent to which \( \hat{D} \) is less than \( C_M \).

Figure 2's smaller checkered area (relative to that of Figure 1) depends upon the 
extent to which the non-market benefits received from EXECON can substitute 
for POLECON's production of \( C \). Let \( D(C(\hat{O}),E(\pi)) \) denote the upper bound on
such substitution. Then the checkered area of Figure 2 is the set of feasible
production optima for Model 2. If non-market benefits do not exist, then (41)
restricts the set of feasible production optima to the frontier segment

\[
S(\hat{D}) \equiv \{ [A(x_A), B(x_B), \hat{D}] : \hat{D} = C(x_C) = D(C(x_C), 0); x_A + x_B + \hat{D} = \} \nonumber
\]

The similarity between Model 1 and Model 2 becomes especially apparent by
expressing Model 2 as a particular form of Model 1:

\[
(47) \quad U[A^0, B^0, D(C(x_C), E(\pi))] = V[A^0, B^0] + \lambda_0 [D - D(C(x_C), E(\pi))] \nonumber
\]

for \( A^0 \geq 0, B^0 \geq 0, x_C \geq 0, \pi \geq 0 \);

\[
(48) \quad U_1[\cdot] = V_1[\cdot], U_2[\cdot] = V_2[\cdot], U_3[\cdot] = \lambda_0 \nonumber
\]

Hence the Kuhn-Tucker conditions for Model 2 only differ from those for Model 1
in the addition of (41) and (44). Thus the Model 2 solution sets for cases
1, 2 and 3 (\( \lambda_2 > 0 \)), denoted by \( T_1, T_2 \) and \( T_3 \), may be constructed directly by
adding (41) to restrictions found in \( S_1, S_2 \) and \( S_3 \) and by substituting \( V_1[\cdot], V_2[\cdot] \)
and \( \lambda_0 \) everywhere that \( U_1[\cdot], U_2[\cdot] \) and \( U_3[\cdot] \) appear. Direct construction yields
the following result where the superscript "+" denotes a case 1 solution, "*"
a case 2 solution and "" a case 3 solution:
Figure 1: Set of feasible production optima for Model 1

Set of feasible production optima is the checkered surface in (A,B,C) space. This set is the same whether or not the non-market benefits exist.

Figure 2: Set of feasible production optima for Model 2

Case when $\hat{D} < C^M$. Set of feasible production optima is the checkered surface in (A,B,C) space. This set is dependent upon the extent to which the non-market benefits can substitute for POLECON's production of C. If such non-market benefits do not exist, then the set of feasible production optima is the frontier segment $S(\hat{D})$. 

$\hat{D} - D\{C(\hat{D}), E(\hat{D})\}$
\[ T^1 \equiv \{ [A(\tilde{x}_A^+), A^0, B(\tilde{x}_B^+), B^0, C(\tilde{x}_C^+), 0] : \frac{A_1(\tilde{x}_A^+)}{B_1(\tilde{x}_B^+)} = \frac{P_B}{P_A} = \frac{V_2[\cdot]}{V_1[\cdot]} ; \frac{A_1(\tilde{x}_A^+)}{C_1(\tilde{x}_C^+)} = \frac{\lambda}{0} \frac{D_1[\cdot]}{V_1[\cdot]} \}; \]
\[ C(\tilde{x}_C^+) = \hat{\tilde{D}}; \frac{V_1[\cdot]}{} > \lambda \frac{D_2[\cdot]}{\tilde{E}_1(0)} \times \tilde{x}_{1A}^+ \geq 0; \times_{1B}^+ > 0; \times_{1C}^+ \geq 0 \quad i = 1, \ldots, N. \]
\[ T^2 \equiv \{ [A(\tilde{x}_A^+), A^0, B(\tilde{x}_B^+), B^0, C(\tilde{x}_C^+), E(\pi^\gamma)] : \frac{A_1(\tilde{x}_A^+)}{B_1(\tilde{x}_B^+)} = \frac{P_B}{P_A} = \frac{V_2[\cdot]}{V_1[\cdot]} \}; \]
\[ \frac{A_1(\tilde{x}_A^+)}{C_1(\tilde{x}_C^+)} = \frac{\lambda}{0} \frac{D_1[\cdot]}{V_1[\cdot]} ; \hat{D} \times C(\tilde{x}_C^+), E(\pi^\gamma) = \hat{D} ; \frac{V_1[\cdot]}{P_A} = \lambda \frac{D_2[\cdot]}{E_1(\pi^\gamma)}; \]
\[ x_{1A}^* \geq 0; x_{1B}^* \geq 0; x_{1C}^* \geq 0 \quad i = 1, \ldots, N. \]
\[ T^3 \equiv \{ [A(\tilde{x}_A^+), A^0, B(\tilde{x}_B^+), B^0, C(\tilde{x}_C^+), E(\pi^\gamma)] : \frac{A_1(\tilde{x}_A^+)}{B_1(\tilde{x}_B^+)} = \frac{V_2[\cdot]}{V_1[\cdot]} \geq \frac{P_B}{P_A} \}; \]
\[ \frac{A_1(\tilde{x}_A^+)}{C_1(\tilde{x}_C^+)} = \frac{\lambda}{0} \frac{D_1[\cdot]}{V_1[\cdot]} ; \hat{D} \times C(\tilde{x}_C^+), E(\pi^\gamma) = \hat{D}; \frac{V_1[\cdot]}{P_A} = \frac{R_B - R_A}{R_B - R_A} = \lambda \frac{D_2[\cdot]}{E_1(\pi^\gamma)}; \]
\[ x_{1A}^* \geq 0; x_{1B}^* \geq 0; x_{1C}^* \geq 0 \quad i = 1, \ldots, N. \]

Cases 1, 2 and 3 for Model 2 are distinguished by the increasing value which non-market benefits received from EXECON have for POLECON. The inequalities describing the solutions for cases 1, 2 and 3 appearing in (33) hold for Model 2 as well, except that \( D[\cdot] = \hat{D} \) for all cases.

The results of Model 2 as compared to Model 1, for identical \( \frac{P_B}{P_A}, \frac{R_B}{R_A}, \tilde{x}_1, A', B', C, D[\cdot], \) and \( E[\cdot], \) exhibit differences for cases 2 and 3. Ignoring the presence of non-market benefits, suppose Model 2's and Model 1's consumption of C is the same, namely \( \hat{D} = D(\tilde{x}_C^+), 0 \). Now let us take into account non-market benefits of a magnitude which will produce new case 2 and case 3 solutions in both models. Each new Model 2 solution relative to its Model 1 counterpart is characterized by greater production and consumption of \( A \) and \( B, \) less production and consumption of \( C, \) and less trade with EXECON.
Section 3: Concluding Remarks

Social conflict encourages government decisionmakers to seek hidden ways to pay for the non-market benefits associated with bilateral agreements. This search has resulted in the presence of unconventional gains from trade, that is, securing these non-market benefits through bilateral preferential trade treatment. The idea that non-market benefits are purchased consciously through bilateral preferential trade treatment provides an explanation for the existence of more than one set of international terms of trade.

The results of Models 1 and 2 indicate that whether POLECON trades with ROW or EXECON or both depends on the extent to which non-market benefits substitute for C and on POLECON's relative preference for C. When making marginal trade decisions POLECON always chooses the greater of world market terms of trade and the marginal terms of trade inclusive of EXECON's non-market benefits. Finally, POLECON's ability to secure unconventional gains from trade increases her utility, and in the case when A, B and C are normal goods, creates optimal patterns of production and consumption which include greater A and B.
Footnotes

1 We are indebted for financial support to the National Council for Soviet and East European Research. We are grateful to David Donaldson, Craig Hakkio, Ronald Harstad and John Pomery for useful comments on an earlier draft, and to Martha Weidner for research assistance.

2 See Marrese and Vaňous [6] for two observable examples of government decisionmakers engaging in preferential bilateral trade in order to purchase non-market benefits.

3 For literature on customs unions, see Kemp [1] and Lipsey [2].

4 This classification is fairly similar to the one proposed by Korbonski [3] who considered the political value to Soviet Union of each Eastern European country as an aggregate sum of its economic, strategic-military, "proxy," and ideological "values" (p. 5).

An alternative framework for explaining how a country with superpower aspirations can use trade policy to advance her political-economic power was developed by Hirschman [1] thirty-five years ago. Hirschman sees foreign trade having two principal effects upon the power position of a country with superpower aspirations. First, Hirschman defines the supply effect to include economic gains from trade that increase the economic power of the dominant country. Second, foreign trade becomes a direct source of power if other smaller countries become economically dependent on the dominant country and thus provide it with an instrument of coercion. This effect Hirschman calls the influence effect. Then the power to interrupt or redefine commercial relations with a dependent country is the root cause of the influence or power position which the dominant country acquires over other nations. The influence effect requires that the dependence of the trade partners on foreign trade must be greater than that of the dominant power. Under such circumstances, dependent countries will likely grant the dominant country certain economic, political, and military advantages in order to maintain stable trade relations. Such dependency is enhanced to the extent that the smaller countries cannot dispense with trade with the dominant country, or replace it as a market and a source of supply. See Marer [5], pp. 27-28.

Also notice that in the case of benevolent interdependent utility functions, purely altruistic motives could lead to preferential trade treatment.

5 An excellent example of the calculus of conventional economic benefits and costs in the case of Soviet-East European economic relations from the Soviet perspective is presented in Marer [5] Sections A and B of the table presented on p. 9a.

6 Even quota rights, preferential (below market) credit arrangements for bilateral trade and preferential tariff treatment can be translated into changes in the terms of trade. Thus they, too, may be viewed as implicit export and import subsidies.
π is defined in terms of a common international currency. Naturally if trade with ROW occurred in one currency and trade with EXECON in another, exchange rate considerations would have to be incorporated in the calculation of π.

We do not discuss the determination of the magnitudes of specific per unit export and import subsidies. As explained in section 1, the crucial assumption from the point of view of section 2 is that the incremental payment for an additional unit of non-market benefits increases as the quantity of non-market benefits.

Varian [7, p. 258].

The set of feasible consumption points is closed and convex due to the non-negativity constraints. The set of feasible production points is closed and bounded since it is defined on a closed set and is subject to weak inequality constraints; moreover, it is convex because the production functions are convex. Each set of feasible trades with each trading group is closed and convex because each balance of trade constraint is an equality condition. Each set of feasible trades is bounded due to the boundedness of the feasible production set. Thus, the constraint set, an intersection of the above sets, is compact and convex.

Varian [7, p. 266].

Varian [7, p. 261].
References


