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NOTE

This is volume 2 of a three-volume report presenting the results of Council Contract No. 800-25 with Princeton University entitled "Economic Disequilibrium and Rationing in East European Countries," Richard E. Quandt, Principal Investigator. Each volume consists of one or more research papers as enumerated below, plus an Executive Summary.

Volume 1:


ESTIMATING THE POSSIBLE SIZE OF PLAN ERRORS, by Richard Portes, Richard E. Quandt, David Winter, and Stephen Yeo

ON THE ESTIMABILITY OF STRUCTURAL PARAMETERS IN ONE-SIDED DISEQUILIBRIUM MODELS, by Richard E. Quandt


Volume 2:

MODELLING PARALLEL MARKETS IN CENTRALLY PLANNED ECONOMIES: THE CASE OF THE AUTOMOBILE MARKET IN POLAND, by Wojciech Charemza, Miroslaw Gronicki, and Richard E. Quandt

ENTERPRISE PURCHASES AND THE EXPECTATION OF RATIONING, by Richard E. Quandt

Volume 3:

BUDGET CONSTRAINTS, BAILOUTS AND THE FIRM UNDER CENTRAL PLANNING, by Stephen M. Goldfeld and Richard E. Quandt
The principal paper consists of a detailed investigation of the Polish automobile market. To the best of my knowledge, this is the first investigation of a durable goods market in an East European economy employing sophisticated econometric techniques.¹

There are two interesting features to this market. First, although it may be debatable whether aggregate consumption always exhibits excess demand, we felt that in the case of automobiles it is safe to model the market for new cars as one always exhibiting excess demand. Secondly, there is not just one automobile market in Poland, but three interrelated ones. There is the market for new cars sold against Polish currency (zlotys) which we assume to exhibit excess demands. Secondly, there is the market for new cars that the state sells against hard currencies; this market is presumably in equilibrium and what adjusts is the black market rate for dollars and other western currencies. Finally, there is the market for secondhand cars which presumably is also in equilibrium. These markets are characterized by spillovers; that is to say, the excess demand in the zloty new car market affects the price of dollars and of secondhand cars. We formulated the model as a 3-equation disequilibrium model and derived the appropriate econometric estimating method. We obtained parameter estimates which were compatible with a priori expectations. Thus, for example, household incomes and the prices of the various types of cars affect demand in the predicted manner. What is even more interesting is that the model can be manipulated to yield endogenous predictions for the queueing time for cars; i.e., for the

¹Of course, there are several papers using essentially descriptive techniques; see, for example, Kapitany, Z., J. Kornai, J. Szabó "Reproduction of Shortage on the Hungarian Car Market," Soviet Studies, XXXVI (1984), 236-256, and Daniel, Z. and A. Semjen, "Housing Shortage and Rents: The Hungarian Experience," (1986) mimeographed.
amount of time that the average person has to wait after ordering a car in
the zloty new car market before delivery actually takes place. In fact,
through stochastic simulations we were able to put a "confidence limit"
around the point estimate of the waiting time. This confidence band overlaps
what anecdotal evidence has claimed to be a mean waiting time on the order
to 2-3 years. We felt that this study, as the previous analysis of aggregate
consumption, has yielded significant new insights into the operation of a
planned economy.

A second short, theoretical, study addresses a class of questions that
have not been satisfactorily answered as yet. The basic paradigm of the
operations of socialist enterprises is the 'soft budget constraint' due to
Kornai. According to this, socialist enterprises have no effective budget
constraint because the state stands ready to make up any revenue shortfall.
The practical consequence is that enterprises, wishing to assure themselves
sufficient raw materials in an uncertain world, demand indefinitely large
amounts of inputs, thus causing excess demand at all times.

One particular mechanism through which this is supposed to occur is
anticipatory buying of inputs. Thus, if a surplus of some input materials
were to occur, at some point in time, enterprises would start hoarding for
the future and the surplus would disappear. This question is investigated
via a simple theoretical model in which firms pay a penalty for not having
enough inputs in the future, but must also pay inventory costs for excess
inputs accumulated in the present. The upshot of the model is that the
expectation that inputs might be rationed in the future does shift some input
purchases to the present, but that under various reasonable scenarios the
magnitude of this effect is fairly small. The Kornai view has to be taken
therefore with a grain of salt.
MODELLING PARALLEL MARKETS IN CENTRALLY PLANNED ECONOMIES:
THE CASE OF THE AUTOMOBILE MARKET IN POLAND

by
Wojciech Charemza
Miroslaw Gronicki
Richard E. Quandt

1. Introduction

Most empirical work dealing with socialist economies appears to be oriented towards macroeconomic problems or at least to the study of highly aggregated time series. Examples are provided by Howard (1976), Lacko (1975), Podkaminer (1982), Portes and Winter (1980), Portes, Quandt, Winter and Yeo (1983, 1984, 1985), Welfe (1983), and Charemza and Gronicki (forthcoming). Only relatively infrequently has a particular market been the target of detailed empirical investigation; a case in point is the study of the Hungarian car market by Kapitany, Kornai and Szabo (1984).

In the present paper we examine the complicated structure of a consumers' durables market on which shortages occur and which generate parallel mechanisms of exchange. Such an undertaking differs markedly from analogous endeavors in the context of free-market economies. Some of the more salient differences are the following. (1) In the free-market context it is commonly assumed that either equilibrium is attained or if not, a continual approach to equilibrium occurs because of a partial adjustment to a discrepancy between desired and actual stocks. In socialist economies prices are controlled and change only infrequently and it is commonly argued that socialist economies invariably exhibit excess demand (Kornai 1980),
Pickersgill (1980), Winiecki (1982), Balicki (1983)), without any tendency to approach equilibrium. Although this proposition is debatable for aggregate demand and supply (Portes and Winter (1980), Portes, Quandt, Winter and Yeo (1983, 1984, 1985)), we consider the assumption of permanent excess demand to be reasonable for particular consumers' durables. (2) The notions of permanent income and rational expectations play a prominent role in free-market approaches to characterizing durables markets (Bernanke (1983)). Whether the change in desired stocks of durables such as cars is proportional to permanent income in socialist economies is certainly debatable. It is also not clear whether the rational expectations view is entirely reasonable in an environment in which central planners can abruptly alter both the lifetime prospects and the transitory component of income. (3) If it is true that there is permanent excess demand, we must ask what theory of consumer behavior is compatible with that state of affairs. According to the Dreze (1975) concept, consumers maximize utility subject to the budget constraint and all quantity constraints simultaneously; hence their effective demands ought not to exceed their allotted rations. It is also not clear whether consumers can be thought of as adjusting their labor supply between work and leisure (and between the 'normal' economy and the 'underground' economy (Charemza and Gronicki (forthcoming)) in response to perceived rationing in commodity markets. We must therefore investigate, at least schematically, what kind of utility function is compatible with a reasonable definition of permanent excess demand. (4) In comparison with free market economies, the data problems are enormous. Even for the car market, for which data seem to be relatively abundant, no data (such as registration or insurance records) are available from which a quarterly series on the stock of cars could be constructed and so no aggregate equations such as those of Chow (1957) can be
estimated. There are no panel data and so procedures such as those of Bernanke (1983) are not possible. In certain submarkets one cannot even ascertain the quantity of cars transacted. Under these circumstances, model formulation and estimation are going to be difficult.

In Section 2 we introduce a simple model of the consumer that is compatible with the stylized facts of the market; this model is applied in the computation of queue-length in the econometric work. In Section 3 we discuss the basic features of the automobile market. In Section 4 we formally specify the model. Section 5 is devoted to problems of estimation and Section 6 contains a discussion of results. Section 7 contains some brief conclusions.

2. A Model of the Consumer

It is a fact of life in socialist economies that certain consumers' durables such as cars are, on the whole, not readily available. Consumers have to queue and this is said to be compatible with the normal state of the market (Kornai and Weibull (1978), Katz and Owen (1984)). An important feature of such a situation is that, at any one moment or over any unit period, the number of consumers requiring service (demand) is greater than the number being serviced (supply); moreover, that this situation can represent an equilibrium in the sense that there are no forces tending to change the queue. We illustrate this with a very simple model, related to that of Lindsay and Feigenbaum (1984), and lacking the elaborateness of Katz and Owen (1984). The role of the model is to provide an interpretation of excess demand for the case of durable goods and to provide the context for the computation of queue length and waiting time in Section 6.
We posit the simple utility function

\[ V(x, y) = U(x) + ye^{-\beta w} \]  \hspace{1cm} (2-1)

where \( x \) is a composite good with price normalized to unity, \( U(x) \) is concave, \( y = 1 \) if the consumer enters the queue for the durable good at the present time and zero otherwise, \( w \) is the amount of time he has to wait for the delivery of the durable good. The parameter \( \gamma \) measures the utility contribution of the durable good relative to the composite good and may assume different values for different consumers.\(^1\) The waiting time transforms future car services into present utility and \( \beta \) acts as the discount rate (for simplicity we assume that \( \beta \) is identical for all consumers). Both \( \gamma \) and \( \beta \) are assumed to be positive. The additive separability of the two types of goods appears reasonable but otherwise the utility function is chosen to express the effect of queuing as conveniently as possible. Given an income \( M \), the budget constraint is

\[ x = M \text{ if the consumer does not enter the durable good queue} \hspace{1cm} (2-2)\]
\[ x = M - p \text{ otherwise} \]

where \( p \) is the price of the durable good paid irreversibly at the time of entry into the queue.\(^2\) The expected utility of the consumer is

\[ U(M) \text{ if he does not enter the queue} \]
\[ U(M-p) + \gamma E(e^{-\beta w}) \text{ otherwise.} \]

He will enter the queue if

\[ U(M-p) + \gamma E(e^{-\beta w}) > U(M). \]  \hspace{1cm} (2-3)

Assume that (1) we have a continuum of consumers, each characterized by a particular value of \( \gamma > 0 \), with density \( h(\gamma) \), and (2) arrivals in the queue and service in the queue are independent with arrival and service rates \( \lambda \) and \( \mu \) respectively (\( \mu > \lambda \)) and that the interarrival and interservice times are
exponentially distributed (which preserves the Markov property of the process). In the present context the arrival rate is the rate at which customers arrive at the queue and the service rate is the rate at which customers in the queue obtain the durable good. Then the density function of waiting time $w$ is

$$f(w) = (\mu - \lambda)e^{-(\mu - \lambda)w}$$

(2-4)

It follows immediately that

$$E(e^{-\beta w}) = \frac{\delta}{\delta + 1}$$

(2-5)

where $\delta = (\mu - \lambda)/\beta$. Then (2-3) becomes

$$Y > \frac{\delta + 1}{\delta} (U(M) - U(M-p)) = Y_0$$

(2-6)

and the fraction of consumers entering the queue is

$$P = \int_{Y_0}^{\infty} h(y)dy = 1 - H(Y_0)$$

(2-7)

where $H$ is the cumulative distribution function of $h$. If $N$ is the total number of consumers, the arrival rate $\lambda$ is $NP$. The arrival rate depends on $P$ and in turn determines $P$ as the outcome of optimal decisions by consumers; an equilibrium exists if the mapping $P \rightarrow P$ possesses a fixed point. The key notion here is that one of the parameters of the queuing, the arrival rate, is endogenous. The arrival rate (as well as the service rate, and the parameters of the utility function) determines the expected waiting time and that influences the results of utility maximization, which in turn determines a new value of the arrival rate. Consumers are in equilibrium if no further adjustment is necessary; i.e. if the expected waiting time "assumed" in the utility maximization exercise yields an arrival rate which implies that same expected waiting time. It is simple to show that
\[ P = 1 - H \left[ \frac{(\mu-NP)/\beta+1}{(\mu-NP)/\beta} (U(M) - U(M-p)) \right] \] (2-8)

has a unique solution for \( P \) in the \( 0 < P < \mu/N \) interval. Then the expected waiting time and queue length corresponding to this solution value of \( P \) are themselves equilibrium values.

From (2-8) we immediately obtain several comparative statics results. Denote by \( P_M, P_p, P_\delta \) the partial derivatives of \( P \) with respect to \( M, p \) and \( \delta \) respectively, and let \( \Delta U \) denote \( U(M) - U(M-p) \). Differentiating (2-8), we obtain

\[
P_M = \left[ \beta \delta^2 + H'N\Delta U \right]^{-1} \beta \delta(\delta+1)H' \cdot [U'(M-p)-U'(M)]
\]

\[
P_p = -\left[ \beta \delta^2 + H'N\Delta U \right]^{-1} \beta \delta(\delta+1)H' \cdot U'(M-p)
\]

\[
P_\delta = \left[ \beta \delta^2 + H'N\Delta U \right]^{-1} \beta H' \cdot \Delta U
\]

\( H', \Delta U \) and \( U' \) are positive and \( U'(M-p) - U'(M) \) is positive (by virtue of the concavity of \( U \)). Hence \( P_M > 0, P_p < 0 \) and \( P_\delta > 0 \). From the latter it follows that an increase in the service rate increases the willingness to enter the queue, whereas an increase in \( \beta \), i.e. the rate at which utility decays with waiting time, reduces the willingness to enter the queue. But no matter how the parameters of the problem change, in equilibrium there will always be a queue, even though more unfavorable circumstances reduce the willingness of consumers to enter the queue.

In the light of the above it is convenient to interpret excess demand in the aggregate time series sense to consist of the difference between the quantity demanded by consumers in the queue on the average in a given time interval and the quantity made available in that time interval. Although in a flow sense there is no excess demand in the queuing model, since the
arrival rate must be less than the service rate, the amount of goods consumers are willing to purchase in any one time interval may be much greater than the amount of goods delivered in that interval. The consumer optimizes on the basis of his income, prices and waiting time and an equilibrium emerges in which the waiting time is endogenous and is jointly determined by the consumer's decision to enter the queue. Although his expectation of long waits reduces his probability of entering, in equilibrium there is no reason for the queue to disappear. "Disequilibrium" is thus somewhat of a misnomer: the lack of equality between demand and supply is an equilibrium property of the system.

3. An Example: The Automobile Market

The markets for cars and dollars (and other Western currencies) are the best known examples of 'parallel' markets in centrally planned economies, especially in Poland. These markets are large and exert a strong influence on smaller legal and illegal fields of private activity. They are also relatively well-organized and statistical data about them, however incomplete, are more plentiful than about other markets. The Polish car market is described in detail by Krasinski, et al. (1980) and by Starzec (1983).

The complicated structure of the car market stems from the fact that various systems exist for conveying new cars from the seller (the state) to consumers. The institutional arrangements provide for different prices and methods of payment in each of the resulting submarkets.

A major distinction is between cars that are on the market for the first time and second-hand cars (though note that this distinction is not the same as that between new and used cars, since some new cars are purchased for
speculative reasons and are immediately resold). The supply of cars sold for the first time is predominantly from the state. In the period under investigation (1973-IV to 1982-IV) the private importation of new and used cars was negligible and amounted to 4.6 percent of deliveries in 1974 and 3.6 percent in 1979 (Krasinski, et. al. (1980)). The rest of the supply came from the state and consists of cars manufactured in Poland, as well as imported. The cars sold by the State are actually sold by state enterprises: primarily by POLMOZBYT, which sells cars for Polish currency (zlotys) and by PEWEX, which sells cars (and other consumption goods) for U.S. dollars and other hard currencies. In the 1974-79 period the percentage of total state deliveries accounted for by dollar sales ranged from 16 to 23 percent (Krasinski, et. al. (1980)).

The official zloty price is paid by consumers to POLMOZBYT in installments. A fraction is due upon entering the queue, another fraction during the waiting period and the balance upon delivery. Delivery of a car occurs after a long period in the queue, which anecdotal evidence reports as measured in years. The price is extremely stable: for the Fiat 125p, for example, it was changed only three times in the 1974-1982 period. This zloty price is invariably the lowest price at which a car can be purchased.

The cars sold by PEWEX were usually available without appreciable queuing and were often of higher quality. Although it is convenient here to pretend that all these sales were against dollars, they were in fact against dollars and some other hard currencies as well as against 'dollar vouchers'. (For a similar practice in Hungary, see Kapitany, Kornai, Szabo (1984).) The legal aspect of these transactions is not straightforward. Polish citizens were allowed to import and possess Western currencies but could legally sell them only at official banks at an official price. Dollar vouchers, on the other
hand could be obtained at banks for dollars at par and could also be freely traded at presumably market clearing prices and also used to purchase cars as PEWEX. The free market for dollar vouchers and the black market for dollars are closely related and in this paper they are regarded as a single market. The prices of dollar vouchers and black market prices for dollars were nearly identical and from the point of view of the potential car buyer it made no difference whether he paid PEWEX in dollars or dollar vouchers. If not imported from abroad, either of these instruments had to be purchased privately. In general, hard currencies played a significant role in private Polish economic activity. It is estimated that U.S. dollars in circulation in Poland ranged from 1.5 to 6 billion dollars (Mojkowski (1984)) and various estimates put the monthly volume of black market hard-currency transactions alone at 2.5-4.2 million dollars. 8

A third market for cars is the second hand market in which cars are sold by private firms specializing in the sale of second hand cars, by advertising and principally through 'car exchanges' organized in large cities as open-air markets. 9 There is reason to believe that prices in the second hand market (in zlotys, dollars or dollar vouchers) were market clearing.

To summarize: there are in effect three markets for cars in Poland. The first is the official zloty market for new cars that does not have a market clearing price. The second is the official dollar market for new cars (PEWEX) that involves shorter queues and higher prices. Finally, there is the second hand market in which queues are negligible. In the latter two, the price presumably does clear the market.

4. The Basic Model

An ideal model would specify demand and supply functions for all three
submarkets and the stock of cars would play an important role in this specification at various points. Unfortunately, such a model would not be practical because key data required by it are not available. For the period investigated, 1973-IV to 1982-IV, we have found no quarterly data on the stock of cars, nor on the quantities of cars traded in the PEWEX market and in the second hand market. In general, one would have to consider this lack a serious disadvantage in attempting to estimate a car model. However, in Poland it appears reasonable to argue that, because of permanent shortage, the stock of cars has never reached a level at which it would exert an appreciable effect on the demand. We therefore content ourselves with an inferior specification in which some of the structural equations are already partially reduced; i.e. they stand between the original structural equations that would emerge from agents' optimizing behavior and the reduced form equations.

Notation. The variables in the econometric model are as follows:

\[D\] = demand for new cars in the zloty market
\[BM\$\] = black market price of U.S. dollars
\[RP\] = retail price index
\[PF\] = zloty price of second hand cars
\[INC\] = nominal personal income
\[DUM\] = the "success illusion" dummy; equals 1 for 1978-1 to 1979-1 and 0 otherwise
\[PO\] = the official zloty price in the new car market
\[PCD\] = the official dollar price of new cars in the PEWEX market
\[CAR\] = quantity of cars (supply) made available to the zloty-denominated new car market
E( ) = expectation of ( )

\[ \xi_1, \xi_2, \xi_3 = \text{normally distributed error terms, independent of one another.} \]

We generally omit the time subscript for simplicity except for the subscripts +1 and -1 to denote leads and lags.

Equations. The jointly determined variables of the model are \( D, BM$/RP \) and \( PF/RP \). We write the three equations by normalizing on these variables in turn as

\[
D = a_0 + a_1 (INC/RP) + a_2 [E(BM$+1/RP) - BM$/RP] + a_3 DUM
+ a_4 (PF/RP - PO/RP) + \xi_1 \quad (3-1)
\]

\[
PCD(BM$/RP) = \beta_0 + \beta_1 (D-CAR) + \beta_2 (PO/RP) + \xi_2 \quad (3-2)
\]

\[
PF/RP = \epsilon_0 + \epsilon_1 (PF/RP)_{-1} + \epsilon_2 (BM$/RP-(BM$/RP)_{-1})
+ \epsilon_3 (PO/RP) + \xi_3 \quad (3-3)
\]

The first of these equations is the demand function for new cars in the zloty market and is a proper structural equation. In view of the previous discussion, it is the demand (per unit period) of all those who receive cars from the state and who are willing to pay zlotys for them, plus those who are still in the queue. A priori we expect \( a_1 > 0 \) (an increase in real personal income increases car demand), \( a_4 > 0 \) (the relative price effect; the greater is the free market price relative to the official new car price, the greater the demand for new cars) and \( a_3 > 0 \). This latter is attached to a dummy variable which takes values of 1 during certain quarters (1978-I to 1979-I) of the Gierek Government when the illusion of success for the Polish economy appeared to be widespread.12 The coefficient \( a_2 \) measures the effect of an expected real increase in the price of dollars. Dollars are not only the way
to buy a PEWEX car but are also a generalized store of value and investment instrument for consumers. The increase in the expected value of dollars may lead to some substitution away from PEWEX cars to zloty cars, and may in particular increase the demand for zloty cars for speculative purposes. At the same time, the expectation of more expensive dollars in the future, will tend to increase the current demand for dollars and, ceteris paribus, reduce the current demand for new cars. On balance, we expect the latter effect to predominate and thus $a_2$ is expected to be negative.\(^ {13}\)

The observed queue length should, ceteris paribus, be included in the demand equation. As we indicated before, the queue length is unfortunately not observable. We attempted a compromise solution by including as a variable the queue length that is implicitly given by the model itself. As was the case with some other attempted extensions of the model, this placed too heavy a burden on it and rendered most coefficients insignificant upon estimation.

The second equation of the model can be thought of as resulting from equating demand and supply of cars in the PEWEX market. Let this demand be given by $D_X = D_X(PCD(BM$/RP$), PO/RP$, D-CAR, INC/RP$)$, expressing that demand depends on own price (recalculated in zlotys through the black market dollar price), the competing product's price, on excess or unsatisfied demand in the competing market and on income. It is our hope that the income variable will capture the motivations to demand dollars that are in addition to those emanating from the car market. It is much harder to describe the supply function. Kapitany, Kornai and Szabo (1984) argue that supply is not affected by price and profitability but by the planners' general social goals, some macrovariables such as trade balance, and by excess demand for the cars. We therefore take supply to be a linear function of excess demand.
in the zloty new-car market plus a random error. Equating demand and supply yields an equation that explains the dollar price of PEWEX cars. Income already affects $D_X$ through $D - CAR$ and we delete it from $D_X$ in the main runs to reduce the number of parameters to be estimated. (In one run, however, we retain INC/RP in the equation.) The "dependent" variable explained by (3-2) is $PCD(BM$/RP)$, i.e. the implicit zloty price of PEWEX cars (in this, however, $PCD$ is exogenous and fixed by the authorities and only $BM$/RP is jointly determined). A priori $\beta_1 > 0$ and $\beta_2 > 0$, since the latter measures the substitution effect of higher prices in the official zloty market, whereas the former measures the spillover effect from that market (i.e., the effect of the excess demand in that market).

The third equation of the model is a partially reduced structural equation that explains $PF/RP$, the market-clearing price in the second hand car market. The equation may again be thought of as resulting from equating demand and supply in that market. Supply in this market is likely to depend on recent past prices and we include $(PF/RP)_{-1}$ as an explanatory variable. If supply responds negatively to lagged price and positively to current price and if demand has an inverse relation to current price, the autoregressive coefficient $\varepsilon_1$ will be positive. The coefficient $\varepsilon_3$ is also expected to be positive. The case of $\varepsilon_2$ is more difficult. It measures the effect of a recent increase in the real price of blackmarket dollars.14 This effect is not fully unambiguous. On the one hand, to the extent that dollars are an important liquid asset, an increase in the value of dollars generates a positive real balance effect. On the other hand, if the demand for dollars is inelastic, an increase in its value generates a negative income effect. A further consideration also suggests a negative effect: in critical times of uncertainty, dollars are a particularly good store of value (easily and
safely stored, etc.) whereas cars are a particularly unsuitable asset (bulky, subject to damage, and potentially unusable because of gasoline restrictions and other reasons). Hence general increases in the value of dollars may be related to lower free market prices. On balance we posit a negative relationship here.

There are a number of alternatives one could consider. Specifically, the level of personal income could reasonably be included in each equation, since the demand for each type of car is likely to depend on it. One might well include in the equations the quantities of other durable good in short supply (e.g., housing) in order to measure the spillovers due to rationing in these markets. The excess demand in the zloty new-car market may spill over into the market for second hand cars (for a similar suggestion see Kapitany, Kornai, Szabo (1984)). Some of these other possibilities are briefly discussed in Section 5. However, we are already estimating 15 parameters (12 \( \alpha \)'s, \( \beta \)'s, \( \epsilon \)'s and 3 error variances) in the basic model with 35 effective data points. Any substantial increase in the number of parameters is likely to create difficulties and we have attempted to stick to as parsimonious a formulation as possible, even if not entirely correct from the theoretical point of view.

The model is a disequilibrium model in that demand for new cars is not assumed to be equal to supply. In fact, we assume that we observe the supply (CAR) and that in every period

\[
D \geq \text{CAR} \tag{3-1}
\]

and that, as usual, the demand D is not observed by the econometrician. This makes for a disequilibrium model that is slightly different from the usual ones. We now turn to the problems of estimation.
5. Methods of Estimation

**Maximum Likelihood.** The generally preferred method of estimation for disequilibrium models is maximum likelihood. The joint density (for observable and unobservable variables) is obtained from the structural equations and the unobservable variable is integrated out over the regions corresponding to the excess demand and excess supply regimes. If no a priori sample separation is given, the density is the sum of the two integrals. Simplifying for the moment by restricting attention to the standard simple disequilibrium model in which there exists only a demand function, supply function and min condition, the density of the transacted quantity can be written as

\[ h(Y) = g_1(Y)(1-G_2(Y)) + g_2(Y)(1-G_1(Y)) \]  

(4-1)

where \( g_1 \) and \( g_2 \) are densities and \( G_1 \) and \( G_2 \) are the cumulative distributions of demand \( D \) (supply) conditional on supply \( S \) (demand) and evaluated at \( Y = \min(D,S) \) (Hartley (1977)). In the present case we made the a priori assumption that only excess demand occurs. (For the derivation, see Appendix B.) This is equivalent to the assumption that the term in (4-1) corresponding to excess supply (say \( g_2 \cdot (1-G_1) \)) is zero, which is identical to the assumption that the conditional probability \( \Pr\{S>D|Y\} = 0 \). The likelihood function is then the product over the observations of terms such as (4-1).

**Estimation of the Condensed Model.** An alternative approach is to condense the model by solving Equ. (3-1) for \( D \) and substituting this in (3-2) (Hartley and Mennemeyer (1974)). This yields an ordinary nonlinear simultaneous equations system and may be estimated by FIML. It can be shown (Quandt (1985), Portes, Quandt, Yeo (1985)) that the density function of the random variables BM$/RP and PF/RP from this condensed model is identical with
that part of the density function derived in Appendix B that corresponds to the term in (B-3) that does not involve the cumulative normal $1 - \Phi(\lambda)$. Note that in the condensed model the variable $D$ is replaced for all its data points, which follows from the assumption that all observations lie on the supply curve and that the probability that $\text{CAR}$ exceeds $D$ is equal to (or close to) one. This indicates that the condensed method is based on a stronger a priori assumption than the disequilibrium FIML method.

Concluding Comments. It is to be noted that in general it is not obvious that the parameters of the function representing the 'long' side of the market are estimable. Sufficient conditions for estimability are not known, although some necessary conditions for estimability are derived in Quandt (1985). These conditions are satisfied for the parameters of the equations. Even then, the variance of the equation may not be identified; this can be seen most clearly by inspection from the condensed model. For this reason, in the condensed procedure we only estimate a combined variance for the first equation. In the disequilibrium maximum likelihood method we arbitrarily fix the value $\sigma^2_1$ at 2.0.

6. Empirical Results

The principal results are displayed in Table 1. Columns are designated by model number and estimation method. $M$ designates the disequilibrium maximum likelihood procedure and $C$ the condensed procedure. Model 1 is the basic one discussed in Section 4 with the proviso that the expectation variable $E(BM_{t+1})$ is replaced by $BM_{t+1}$, i.e., perfect foresight is assumed. In this model we obtain the real expected future price of dollars by deflating by $RP$. In a variant, we deflate by $RP_{t+1}$, which implies that the real value of dollars is foreseen perfectly. The results of this variant
# TABLE 1. Results of Estimation

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<th>1/M</th>
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<th>3/M</th>
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<td>(3.187)</td>
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<td>( \log L )</td>
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<td>42.679</td>
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<td>41.157</td>
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</table>

*t-ratios are given in parentheses below the estimates.
are extremely similar to those of Model 1 and are not reported explicitly. In Model 2, the variable INC/RP in the demand for new cars is replaced by DEP/RP, where DEP represents the end-of-quarter deposits of households in savings institutions. In Model 3 we modify Model 1 by including the spillover term $\varepsilon_4(y_1 - \text{CAR})$ in the equation explaining the price of second-hand cars. In Model 4 we include in this equation the term $\varepsilon_4(\text{INC/RP})$.

The parameter values have the expected pattern of signs in Model 1/M. Personal income is not significant in Equ. (3-1), although the other coefficients in that equation and in Equ. (3-2) are significant. The only significant coefficient in the second-hand car price equation is that of the lagged price although all coefficients at least have the expected sign. In particular we note that the spillover from the normal (zloty) new-car market to the PEWEX car market ($\beta_1$) is positive and significant, that the effect of an expected dollar appreciation ($\alpha_2$) has a significant negative effect on the demand for new zloty cars, and that a recent past appreciation of the dollar ($\varepsilon_2$) has a nonsignificant negative effect on the free market price. The insignificance of this coefficient may well reflect the conflicting influences that the right-hand-side variable was expected to have on the free market price.

In Model 3 we also include the spillover from the new zloty market in Equ. (3-3). The effect is a nonsignificant increase in the loglikelihood and a numerically very small measured positive spillover effect; we thus have no reason for preferring model 3/M.

Model 1/M and 1/C are directly comparable. The likelihood function values are essentially identical and all coefficients except two of the constant terms are essentially identical. This underscores the similarity of the respective likelihood functions and is compatible with the fact that the
estimates in 1/M imply a value near 1 for $1-\phi(\cdot)$ in the density function (B-3). Model 1/C does a worse job, however, of estimating the constant terms and the resulting excess demand predictions are less plausible.

We also estimated some variants of Models 2/M. Since they use different variables, they are not nested in 1/M and likelihood value comparisons are not particularly meaningful. Their coefficient estimates are broadly similar to those of 1/M without any particular reason for preferring them to 1/M.

We estimated a model similar to Model 3/M from which, however, $\varepsilon_3$ was excluded; it did not differ from Model 3/M materially. We also estimated a model in which the perfect foresight of BM$_{t+1}$ was replaced by an ARMA(4,4) prediction: this left the estimates of Equs. (3-2) and (3-3) largely unchanged but made the coefficients of the new car demand completely insignificant.

We finally examined whether it is a serious misspecification not to include INC/RP in Equs. (3-2) and (3-3). Its inclusion in (3-2) yielded a t-value of 0.08 and an increase in the likelihood only in the 5th place. However, its inclusion in Equ. (3-3) (Model 4/M) with coefficient $\varepsilon_4$ yielded a significant coefficient and the likelihood ratio comparison with Model 1/M is also significant at the 0.01 level. We conclude that Model 4/M appears to be the most reasonable one. Our further discussion is based on this model and its comparison to Model 1/M.

Setting the error terms equal to zero, the model can be solved simultaneously for the endogenous variables and thus it can be used to make a within-sample prediction of demand $\hat{D}$. Then $\hat{D} - \text{CAR}$ can be interpreted as the excess demand. Both CAR and this predicted excess demand are displayed in Table 2.15 Returning to our queueing interpretation of Section 2, we attempt to compute the mean waiting time for new cars. For this purpose we
assume as an approximation that the total queue length is the sum of the excess demand and the number of customers actually serviced; i.e., CAR. Denote the sum of these by \( Q \) (queue) and further assume that in any quarter the queue length is the expected queue length. Now \( E(Q) = \rho/(1-\rho) \) where \( \rho = \lambda/\mu \) is defined as the "traffic intensity"; equating \( \rho/(1-\rho) \) to \( Q \) allows us to compute \( \rho \), and since we know the service rate (CAR), we can compute the arrival rate \( \lambda \). Finally, we obtain the mean waiting time as \( Q/\lambda \) (Phillips, Ravindrau, Solberg (1976)). These estimated mean waiting times are displayed in the next to last column of Table 2. Models 1/M and 4/M tell rather similar stories. During the early period, i.e., from 1974-1 to sometime in 1979, excess demand as a percentage of car deliveries ranged from 50.1 to 124.0. At the same time, the mean waiting time was remarkably steady, ranging from 1 1/4 quarters to just over two quarters. This is substantially shorter than anecdotal evidence would have it. It is clear that this computation gives only a point estimate for the mean waiting time and it is of interest to compute an upper boundary of an interval estimate (see below). One also notes that in 1980-82 there begins a sharp increase in the mean waiting time, relieved only temporarily in 1981-1, and reaches the 4 1/4 to 6 quarter range in the 1981-3 to 1982-2 period, followed in the last quarter for which we can make a forecast by a sharp drop in both excess demand and mean waiting time. This final drop may well be due to the existence of gasoline rationing. In general, the sharp increase in waiting times in 1980-82 appears to be correlated with the flagging performance of the Polish economy in this critical period.

To obtain an upper "confidence limit" for the mean waiting time, we resorted to stochastic simulations of Model 4/M. The estimated coefficients of the equations were taken to be the mean value and the estimated
TABLE 2. Excess Demands and Waiting Times

| CAR | Excess Demand | Mean Waiting Time | Mean Waiting Time
<table>
<thead>
<tr>
<th></th>
<th>1/M</th>
<th>4/M</th>
<th>1/M</th>
<th>4/M</th>
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<td>1.63</td>
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asymptotic covariance matrix of the estimates was taken to be the covariance matrix of a normal distribution. Coefficients were generated by drawing from this distribution. Equation errors $\xi_1, \xi_2, \xi_3$, distributed normally with zero mean and variances equal to those estimated from the model, were then added and the mean waiting time for each time period was obtained from the solution of the model. This experiment was replicated 100 times. For each quarter we then determined the 90th percentile of the distribution of waiting times. These are displayed in the last column of Table 2. The interpretation of these figures is the same as that of conventional confidence intervals: for example, using a 0.1 level of significance, we cannot reject the claim that in 1982-1 the mean waiting time was 12 quarters. There are two particularly noteworthy things about this last column of Table 2. First, many of the mean waiting times are quite compatible with the anecdotal evidence of 2-3 years, and none is less than a year. Secondly, whereas the point estimates show relatively homogeneous behavior in the 1970's, the last column shows waiting times in 1974 and early 1975 about as high as in the critical 1981-82 period.

We finally examine the implied price and income elasticities of the exogenous variables from 4/M. Evaluating the elasticities of demand for new cars with respect to (real) income from equation (3-1) yields income elasticities ranging from 0.28 at the beginning of the period to 0.06 near the end. The corresponding price elasticities range from -4.64 to -0.65. Alternatively, one may obtain the elasticities from the reduced form multipliers. These income elasticities range from 0.03 to 0.12 and the corresponding price elasticities from -3.25 to -0.54. The key observation is that the income elasticity of demand for new cars is very small, whereas the price elasticity is quite substantial, in fact, greater than unity in
absolute value for almost half the period. If we consider the variant of the model in which income is included in the second equation, the income elasticity rises slightly, but still remains low.

The stochastic simulations can also be used to obtain "confidence limits" for these elasticities. Since the point estimates of the income elasticities are very low, we are interested in the upper 90 per cent confidence limit which ranges from a low of 0.138 to a high of 0.694 with no noticeable trend except for generally low values in the last 5 quarters. In the case of the price elasticities, the point estimates are increasing over the period. Here we are interested in the lower 10 percent confidence limit which varies from -1.098 to -5.859. The stochastic simulations therefore are compatible with a small but nonnegligible and almost trendless income elasticity and a sizeable and slightly increasing price elasticity.

7. Concluding Remarks

It seems that parallel markets in centrally-planned economies can be subjected to econometric analysis in the future. However, the methods employed may be slightly different from the traditions of applied econometrics. First of all, data are difficult to obtain and in many cases are subject to severe approximations. Consequently, the results have to be treated with additional caution. Secondly, estimation techniques are also different from those applied for models of markets in a more flexible economy. The 'all-excess-demand' hypothesis and permanent unobservability of some variables require non-trivial estimation algorithms. In practice almost every model requires its own specific method of estimation. Problems of specification and estimation are therefore closely related. Unlike the case of a genuine market, in a centrally-planned economy price is not sufficiently
flexible to indicate the direction and the strength of the excess demand changes. The estimation problem becomes more complicated, since one cannot use prices as disequilibrium indicators. Finally, the specification of the model must be sufficiently complex as well as sufficiently flexible so as to accommodate the problems of data availability, (2) cope with the problem of exhibiting the relation between the 'first' and 'second' (and in some cases 'third') markets.

Our results indicate that disequilibrium econometrics can provide insights into problems of this kind. The results on particular car markets are largely in agreement with a priori expectations as well as with some anecdotal evidence. In particular, the estimated fall in excess demand and waiting time at the end of 1982 seems to be a good forecaster of events in the out of sample years 1983-1985, in which the free market car price declined (mainly because of reduction in demand) and waiting time for the officially delivered cars was substantially reduced.
Appendix A. Sources and Construction of Data

The following are the principal sources of data:


**BS:** Biuletyn Statistyczny GUS, Warsaw, 1974-82.

**CTPM:** Official catalogues of POLMOZBYT, 1974-82.

**CTPX:** Official catalogues of PEWEX, 1974-82.

**PCY:** Pick's Currency Yearbook, Pick's Publ., 1975-79.

**PT:** Przeglad Techniczny, Warsaw, 1983.

**QRP:** Quarterly Review for Poland, Economist Intelligence Unit, London, 1982.

**RS:** Rocznik Statystyczny GUS, Warsaw, 1974-82.


**VE:** Veto, Warsaw, 1982-83.

**ZG:** Zycie Gospodarcze, Warsaw, 1974-82.

All the variables were constructed from raw data. The methods of construction required numerous adjustments to yield consistent data series, since in several cases the same source of raw data was not available for the entire period. We now list the principal methods of construction and difficulties with the various data series.

**BM$:** Black market price of dollars computed as a moving average from end of period figures in PCY for 1973-4 through 1979-4. For 1980-1 to the end the data came from several sources, principally from PT, No. 12, 1983, p. 26. The annual data in the latter were converted to quarterly by adjustments based on QRP and VE.
CAR: Deliveries of new cars to official zloty market. The sources are BS and RS. In case of disagreement we took the more recently published figures. In some cases of overlapping annual and quarterly data, we regarded the former more accurate and adjusted the quarterly data to agree with the annual totals.

INC: Households' total personal income in billions of zlotys. The annual figures in RS were adjusted to yield quarterly figures on the basis of the monthly figures in BS for the principal components.

PCD: Index of the official Fiat 125p price (in U.S. dollars) from CTPX (1974 = 100.0).

PF: Index of free market price of cars (1974 = 100.0). This was obtained from the quarterly average prices for seven different types and ages of cars reported in ZG by computing their principal components. Only the first component accounting for 98.9% of total variance was used.

PO: Index of the official zloty price for the Fiat 125p (1974 = 100.0) from CTPM.

RP: Index of retail prices. Published quarterly in BS up to the end of 1975. After that the annual price index from RS was adjusted to provide quarterly interpolations. This relied on quarterly retail sales and on quarterly household expenditures.

DEP: Household's deposits in savings institutions. Sources are RSF, BS, RS.
APPENDIX B. Derivation of the Likelihood Function

The derivation becomes more compact if we introduce the following notation: \( y_1 = D, y_2 = BM$/RP, y_3 = PF/RP, z_1 = a_0 + a_1(INC/RP) + a_2E(BM$+1/RP) + a_3DUM - a_4(PO/RP), z_2 = B_0 + B_1CAR + B_2(PO/RP), z_3 = \varepsilon_0 + \varepsilon_1(PF/RP)-1 + \varepsilon_2(BM$/RP)-1 + \varepsilon_3(PO/RP). \) The equation system then is

\[
\begin{align*}
y_1 + a_2y_2 - a_4y_3 & = z_1 + \xi_1 \\
-\beta_1y_1 + PCDy_2 & = z_2 + \xi_2 \\
-\varepsilon_2y_2 + y_3 & = z_3 + \xi_3
\end{align*}
\]

Assume now that the error terms \( \xi_1, \xi_2, \xi_3 \) are normally distributed with zero means and covariances and with variances \( \sigma_1^2, \sigma_2^2, \sigma_3^2 \). The joint density of the endogenous variables for period \( t \) then is

\[
f(y_1t, y_2t, y_3t) = \frac{1}{(2\pi)^{3/2}\sigma_1\sigma_2\sigma_3} \exp\left\{ -\frac{1}{2} \left[ \frac{(y_1t + a_2y_2t - a_4y_3t - z_1t)^2}{\sigma_1^2} \right. \right. \\
+ \frac{(-\beta_1y_1t + PCDt y_2t - z_2t)^2}{\sigma_2^2} \left. \left. + \frac{(-\varepsilon_2y_2t + y_3t - z_3t)^2}{\sigma_3^2} \right] \right\}
\]

(B-1)

On the assumption that demand always exceeds supply (CAR), the pdf of the observable random variables is

\[
h(y_2t, y_3t) = \int_{\text{CAR}_t} f(y_1t, y_2t, y_3t) dy_1t 
\]

(B-2)

Define

\[
w_{1t} = a_2y_2t - a_4y_3t - z_{1t} \\
w_{2t} = PCDt y_2t - z_{2t}
\]
\[ A_t = \frac{(-\varepsilon_2 y_{2t} + y_{3t} - z_{3t})^2}{\sigma_3^2} \]

\[ B_{1t} = \frac{\sigma_2^2 w_{1t} - \sigma_1^2 \beta_1 w_{2t}}{\sigma_2 + \beta_1^2 \sigma_1^2} \]

\[ B_{2t} = \frac{\sigma_2^2 w_{2t} + \sigma_1^2 w_{1t}}{\sigma_2 + \beta_1^2 \sigma_1^2} \]

By completing the square on \( y_{1t} \) and integrating, we obtain

\[
\hat{h}(y_{2t}, y_{3t}) = \frac{i \text{PCD}_t - B_1(\varepsilon_2^2 \alpha_4^2 - \alpha_2^2)}{2\pi \sigma_3(\sigma_2^2 + \beta_1^2 \sigma_1^2)^{1/2}} \exp \left\{ - \frac{1}{2} \left[ \frac{\sigma_2^2 + \beta_1^2 \sigma_1^2}{\sigma_2^2 + \beta_1^2 \sigma_1^2} \right] \left( B_{2t}^2 - B_{1t}^2 + A_t \right) \right\} \times \\
\times \left[ 1 - \Phi \left( \frac{\text{CAR}_t + B_{1t}}{\sigma_1 \sigma_2 / (\sigma_2^2 + \beta_1^2 \sigma_1^2)^{1/2}} \right) \right] \quad (B-3)
\]

where \( \Phi(\cdot) \) is the cumulative standard normal distribution function. The likelihood function is

\[
L = \prod_t \hat{h}(y_{2t}, y_{3t}) \quad (B-4)
\]

It can be shown by the same technique that if the spillover term \( \varepsilon_4(y_{1t} - \text{CAR}_t) \) is included in Equ. (3-3), the density function becomes

\[
\hat{h}(y_{2t}, y_{3t}) = \frac{i \text{PCD}_t (1 + \varepsilon_4 \alpha_4^2) - \beta_1 (\varepsilon_2 \alpha_4 - \alpha_2)}{2\pi (\sigma_2^2 + \beta_1^2 \sigma_1^2 + \varepsilon_4^2 \sigma_2^2)^{1/2}} \times \\
\exp \left\{ - \frac{1}{2} \left[ C_0(C_{2t}^2 - C_{1t}^2) \right] \right\} \left[ 1 - \Phi \left( \frac{\text{CAR}_t + C_{1t}}{1/C_0} \right) \right]
\]

where
\[ C_0 = \frac{\sigma_2^2 \sigma_3^2 + \beta_1^2 \sigma_1^2 \sigma_3^2 + \epsilon_4^2 \sigma_1^2 \sigma_2^2}{\sigma_1^2 \sigma_2^2 \sigma_3^2} \]

\[ C_{1t} = \frac{\sigma_2^2 \sigma_3^2 \lambda_{1t} - \sigma_1^2 \sigma_3^2 \lambda_{2t} + \sigma_1^2 \sigma_2^2 \epsilon_4 \sigma_3^2 \lambda_{3t}}{\sigma_2^2 \sigma_3^2 + \beta_1^2 \sigma_1^2 \sigma_3^2 + \epsilon_4^2 \sigma_1^2 \sigma_2^2} \]

\[ C_{2t} = \frac{\sigma_2^2 \sigma_3^2 \lambda_{1t} + \sigma_1^2 \sigma_3^2 \lambda_{2t} + \sigma_1^2 \sigma_2^2 \epsilon_4 \sigma_3^2 \lambda_{3t}}{\sigma_2^2 \sigma_3^2 + \beta_1^2 \sigma_1^2 \sigma_3^2 + \epsilon_4^2 \sigma_1^2 \sigma_2^2} \]

where \( \lambda_{1t} \) and \( \lambda_{2t} \) are as before and

\[ \lambda_{3t} = \epsilon_2 \lambda_{2t} + \lambda_{3t} - z_{3t} \cdot \]
FOOTNOTES

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1. We would gain some generality at the cost of a substantial increase in complexity if we write the utility function as U(x) + γf(w), with f' < 0, f'' > 0.

2. This is a reasonable simplification of reality, even though in practice only a fraction of the price need be paid upon entering the queue. However, allowing for a sequence of payments would unnecessarily complicate the model.

3. We take the expectation unconditionally (i.e. not conditioned by queue-length) since we are more interested in the case in which this variable is not observed. This is because order books for cars in Poland have never been published to our knowledge and we conjecture that they have not even been aggregated. For a more detailed but similar development see Katz and Owen (1985).

4. The following fixed-point argument establishes the existence of a unique equilibrium. Consider the two sides of Equ. (2-8) as functions of P. The left hand side is a 45° line through the origin. The right hand side has value 1 - H[(u/μ+1)(U(M)-U(M-p))/(u/μ)] at the origin and has a negative derivative with respect to P. Hence a unique intersection exists in the 0 < P < 1 interval. Furthermore, the limit of the right hand side as P → μ/N is 0; hence μ/N must exceed the solution value. This then also assures that λ < μ at equilibrium, as required.
5. In 1972-82 the cars produced domestically were mainly FIAT 125p, FIAT 126p, SYRENA and POLONEZ (since 1978). The import of new cars accounted for 10-30 percent of total deliveries and consisted mainly of SKODA (from Czechoslovakia), TRABANT (from the GDR), and ZAPOROZEC (from the USSR).

6. Besides PEWEX there exists some other enterprises authorized to sell cars for dollars but are of minor importance.

7. The basic price was subject to some other types of variation. Sometimes it was combined with a lottery for a place in the queue. At other times (1979-80) an 'express price' was introduced which was substantially higher than the basic price but in effect eliminated queuing. There also existed a combined dollar-zloty price as well as a system of coupons to avoid queuing that was available to privileged groups of consumers. (Krasinski, et. al. (1980)).

8. Starzec (1983) suggests that dollars are brought to Poland by tourists, diplomats, Polish citizens temporarily abroad (seaman, etc.); to this we may add undetermined amounts of remittances from relatives in Western countries. It should be noted that there is some disagreement about what the facts are: Simon (1982) estimates private hoards of U.S. dollars in 1980 as no greater than 300 million. However, all sources seem to stress that hard currencies represent a substantial part of the entire stock of money in private hands.

9. As indicated earlier, second hand cars are not necessarily used cars in that a new car purchased from the state could be immediately resold at a profit in the second hand market.

10. Although an annual series on the stock of cars does exist, we did not feel confident about attempting a quarterly interpolation of the series.
11. For a more detailed definition of variables see Appendix A.

12. The substantial investments made by the Gierek government in the first half of the 1970's had backfired for a variety of reasons, including bad harvests, and led to increasing foreign trade deficits and attempts by the government to raise food prices. In 1978, a significant propaganda effort was undertaken to justify past policies by pointing to the increase in consumer goods. See Montias (1982).

13. Note that $E(BM_{t+1}/RP)$ is an expectation of a future value and in conformity with frequent practice is taken to depend only on past, hence predetermined, values.

14. Unlike Equ. (3-1) where the blackmarket price of dollars was intended to capture in part a speculative effect, here we are intending to capture a current effect; hence the use of a current difference rather than a leading difference.

15. The "predicted" values of BM$/RP$ and PF/RP track the actual values tolerably if not very well: for these two variables the $R^2$'s obtained by regressing predicted on actual values are 0.51 and 0.43 respectively.

16. These elasticities are obtained for each year by using the actual values of the predetermined variables for that year and computing the predicted value of $[ay_1/a(INC/RP)][(INC/RP)/y_1]$, etc.
REFERENCES


ENTERPRISE PURCHASES AND THE EXPECTATION OF RATIONING*

by

Richard E. Quandt

1. Introduction

It is the conventional view that socialist planned economies invariably find themselves in a state of excess demand. The putative sources of this condition are numerous and include Kornai's [1979, 1980, 1985] theory of the 'soft budget constraint'. An important corollary of this is the notion of anticipatory purchases by state enterprises. According to this notion, managers generally expect to be rationed in their demands for inputs; hence a current potential excess supply of goods needed for input immediately disappears because managers, fearful of a future shortage, buy up all existing stocks of the good in question. Hence, even if by accident the availability of these goods were such as to be more than enough for current production needs, no excess supply would exist, since any excess would be bought up to insure against possible future shortages.

In the present paper we formulate a simple model that is intended to capture the essence of this mechanism and to suggest the order of magnitude of this effect. The approach is in the general class of inventory models and thus bears some resemblance to other models of this kind, e.g., the Baumol [1952] and Tobin [1956] models of the demand for cash.

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2. The Model

Assume that the enterprise is required to deliver at the end of a period an output $Q$. The input needed for this is given by

$$y = f(Q) + \varepsilon$$

where $\varepsilon$ is an error term with density $h(\varepsilon)$ and $E(\varepsilon) = 0$. The input $y$ may be purchased at the beginning of the period and stored until the end or, if available, it may be purchased at the end. In any event, at the end of the period the input acquired on the two dates together is instantaneously transformed into output. Any number of units $y_0$ may be bought at the beginning of the period and $y_1$ units are acquired at the end. We assume that the amount made available at the end, $z$, is a random variable with density $g(z)$ and support $(0,B)$. Then $y_1 = \min (z, f(Q) + \varepsilon - y_0)$. We also denote $f(Q)$ by $f$ for simplicity.

Costs attributable to purchase arrangements are assumed to be quadratic in the amount by which the enterprise is rationed and, at least initially, linear in inventories. Then costs are

$$c_R(f+\varepsilon-y_0-y_1)^2 + c_1y_0 \quad \text{if } y_1 < f + \varepsilon - y_0$$
$$c_1y_0 \quad \text{if } y_1 = f + \varepsilon - y_0$$

To simplify, assume now that $g(z)$ is uniform over $(0,B)$ and $\varepsilon$ is uniform over $(-\alpha, \alpha)$. Then

$$E(C) = \int_{-\alpha}^{\alpha} \int_{-\alpha}^{\alpha} \left[ c_1y_0 + c_R(f + \varepsilon - y_0 - z)^2 \right] \frac{1}{B} \frac{1}{2\alpha} dzd\varepsilon +$$
$$+ c_1y_0 \int_{-\alpha}^{\alpha} \left[ 1 - \frac{(f+\varepsilon-y_0)}{B} \right] \frac{1}{2\alpha} d\varepsilon$$

Differentiating with respect to $y_0$ and setting equal to zero yields the
beginning-of-period purchase rule\(^1\)

\[ y_0 = f - \left[ \frac{Bc_I}{c_R} - \frac{\alpha^2}{3} \right]^{1/2} \]

if \( f - \left[ c_I B/c_R - \alpha^2/3 \right]^{1/2} \) is > 0 and \( y_0 = 0 \) otherwise.

As is sensible, the early purchase (\( y_0 \)) is the greater, the greater the probability of a shortfall, the smaller the inventory carrying costs and the greater the cost of a shortfall. We also note that \( y_0 \) increases with the uncertainty of the output as measured by \( \alpha \).

Denote the fraction of the required amount of input that is purchased early (\( y_0/f \)) by \( R_0 \) and the probability of a shortage (\( \min (f/B,1) \)) by \( P \). Assuming that the output uncertainty is negligible, we have

\[ R_0 \approx 1 - \left( \frac{c_I}{c_R} f P \right)^{1/2} \]

\( R_0 \) depends explicitly on \( f \), though obviously not on the units in which it is measured. But the dependence of \( R_0 \) on \( f \) makes the numerical value difficult to interpret, particularly because the appropriate magnitude of \( c_I/c_R \) is not obvious. The elasticity, \( P dR_0/R_0 dP \), seems more meaningful and is fairly small: for \( f = 100, P = 0.5 \) and \( c_I/c_R \) ranging from 0.1 to 10.0, the elasticity is between 0.022 and 0.404.

3. Conclusion

The expectation of future rationing does shift some input purchases to the present. However, the magnitude of the effect is moderate and the elasticity of the fraction of purchases made earlier with respect to the probability of future rationing is fairly small. Although more complicated

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\(^1\)The second derivative is positive for \( f > y_1 \).
cases may show a greater variety of results, it does not seem plausible to argue that the expectation of future rationing causes all current excess supplies to disappear.

References


Kornai, J., Economics of Shortage, Amsterdam, North-Holland, 1980.
