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NOTE

This is volume 3 of a three-volume report presenting the results of Council Contract No. 800-25 with Princeton University entitled "Economic Disequilibrium and Rationing in East European Countries," Richard E. Quandt, Principal Investigator. Each volume consists of one or more research papers as enumerated below, plus an Executive Summary.

Volume 1:
ESTIMATING THE POSSIBLE SIZE OF PLAN ERRORS, by Richard Portes, Richard F. Quandt, David Winter, and Stephen Yeo
ON THE ESTIMABILITY OF STRUCTURAL PARAMETERS IN ONE-SIDED DISEQUILIBRIUM MODELS, by Richard E. Quandt

Volume 2:
MODELLING PARALLEL MARKETS IN CENTRALLY PLANNED ECONOMIES: THE CASE OF THE AUTOMOBILE MARKET IN POLAND, by Wojciech Charemza, Mirosław Gronicki, and Richard E. Quandt
ENTERPRISE PURCHASES AND THE EXPECTATION OF RATIONING, by Richard E. Quandt

Volume 3:
BUDGET CONSTRAINTS, BAILOUTS AND THE FIRM UNDER CENTRAL PLANNING, by Stephen M. Goldfeld and Richard E. Quandt
Motivated by the notion of the soft budget constraint, we develop two models of an enterprise that can secure bailouts from the state when its operating profit is negative. The models allow for uncertainty both in production and in the extent to which the enterprise can obtain subsidies. We derive the optimal factor inputs and show, both via theorems and illustrative calculations, how the availability of bailouts increases the demand for factor inputs. The dependence of this increase on the features of the bailout process is also characterized.
1. Introduction

The conventional interpretation of the macroeconomic scene in socialist economies is that they experience chronic shortages of goods and labor. The most eloquent statement of this view is that of Kornai (1980), although it is expressed, interpreted and sometimes amended by numerous students of socialist economies (Gomulka (1985), Podkaminer (1985), Kapitány, Kornai, Szabó (1984), Marrese and Mitchell (1984), etc.). While the chronic shortage view has undoubtedly provided many valuable insights into the workings of centrally planned economies, it is not without critics, even among Eastern European economists (Sóos (1985)). An alternative contrary view is expressed in a series of papers that argue that both in principle and in practice socialist economies are capable of experiencing aggregate excess supply conditions (Portes and Winter (1980), Portes, Quandt, Winter and Yeo (1985, 1987)), not only excess demand.

Our purpose here is not to debate the empirical evidence about centrally planned economies. Rather, we wish to discuss some formalized models of the enterprise that incorporate some of the key aspects of the chronic shortage theory and explore the implications of these models for the demand for inputs. To our knowledge, such formalizations are rare in the literature, with Kornai and Weibull (1983) providing an exception. Finally, we note that the approach
we take is also suited for discussing the problems of firms in free-market economies.

The key question is what happens to a firm when its receipts from the sale of its output do not cover its outlays on inputs. In the free market case, persistent losses are incompatible with survival. In the planned-economy case, it is said, there exist well-established avenues for securing bailouts from the state and the survival of the firm is not threatened, even if it makes persistent losses. To the extent that these bailouts are expected, their existence must be taken into account when the firm's plans are made and thus will affect production schedules and firm's demand for inputs. Insofar as the possibility of bailouts substantially increases the firm's input demand, chronic input shortages may develop. In this paper, we focus primarily on the mechanisms by which the possibility of bailouts affects input demand.

In Section 2 we briefly explore the basis of the chronic shortage view and its implications for a formalized model of the enterprise. Section 3 contains the formulation of our models and the principal implications. Section 4 contains some numerical implications derived on the basis of some simplifying assumptions. Section 5 contains brief conclusions and an agenda for future work.

2. The Soft Budget Constraint

A cornerstone of Kornai's (1979, 1980, 1983) theory of chronic shortage in socialist or centrally planned economies is the idea of the soft budget constraint. The notion is applied to the enterprise or firm in central planning, not to the household, and it is argued that unlike the case of free-market economies, in which the threat of bankruptcy imposes a certain market discipline, the manager in a centrally planned enterprise need not fear bankruptcy. If his outlays exceed his revenues, the state stands ready to make
up the difference. This leads to an insatiable hunger for current inputs, including labor, as well as for investment.¹

The softness of the budget constraint arises from several sources (Kornai (1980, 1983)). (1) Firms are not price-takers; they can thus increase their revenues by increasing prices. This is perhaps the weakest point. It is obviously true that revenues can be increased by raising prices if there already is excess demand; but then the ability to do so can hardly explain the existence of the shortage. It is also not clear what to make of the empirical fact that prices of consumers’ durables change infrequently in various centrally planned economies (Charemza, Gronicki, Quandt (1987)). (2) Taxes are levied flexibly on enterprises. (3) Enterprises may receive gifts from the state. (4) Loans may be made available in circumstances where ordinary considerations of credit-worthiness would argue against them. (5) Other external sources of investment financing may be freely available.

Points (2) through (5) are certainly not unfamiliar in the free-market context. One need only think of the examples of Lockheed or Chrysler to realize that the hardness or softness of budget constraints is a matter of degree only and that some degree of softness is a pervasive phenomenon irrespective of the overall formal organization of the economy. Kornai (1980) notes this and observes that the trend in capitalist economies is in the direction of softening the budget constraint (p.311). But the parallels between the two types of economies depend not only on whether bankruptcies in free-market economies are more frequent than in centrally planned ones.

¹"...we say that the demand for investment goods is almost insatiable." (Kornai (1983) p.51, italics in the original). "...the input demand of firms is almost insatiable.... It can count on sooner or later covering the expenditures on the inputs; if not out of the revenues derived from the sale of outputs, then from some other external financial source". (Kornai (1983), p.83)); (our translation, italics in the original). Also, in (1979, p.809) we have "Demand is not simply too large, but as a first approximation can be formulated as infinite".
Even if that is the case, the important question is what happens to the real resources controlled by the enterprises. They tend to become unemployed (machines junked, factory buildings razed, employees permanently furloughed) much less frequently than the rate of bankruptcies itself. Bankrupt firms sometimes get shots in the arm from new creditors or new owners; assets are sold off, workers stay in their old jobs or move to new ones. What does happen is that current owners and current managers often end up being losers.2

Similarly in central planning, we expect ceteris paribus that managers of enterprises that are chronically bankrupt in the conventional sense do not fare as well as their more successful counterparts.

This raises the question of how a more formal theory ought to account for the motivation of managers in the presence of soft budget constraints. Much of the literature on managers in central planning is rather informal on this subject (for a formal approach see Kornai and Weibull (1983)). Kornai (1980, pp.61-63) argues that managers behavior is strongly affected by their "identification with their own job", by their desire to ensure the "subsistence, survival and viability" of their enterprise and by their desire for a "smooth working process". It is strongly argued that profit maximization plays no role: "the most important standard factors: the price and the expected enterprise profit have no affect on the seller's supply" (Kapitány, Kornai, Szabó (1984)). This latter point is echoed by Podkaminer (1986) who, however, immediately qualifies his agreement: "This is not to say that in the centrally planned economies the enterprises may not -- under some systemic reform -- change their behavioral [sic] pattern so as to resemble more closely the enterprises performing in the market economies." Kornai (1979, p.807) himself

2This is indeed an important point made by Gomulka (1985, p.5) who notes that socialist enterprise managers cannot aim at indefinitely large accumulations of inputs without running the risk of being fired or demoted.
argues that the profit incentive is not incompatible with the soft budget constraint.

In formulating our models we shall work with a stochastic production function and shall adopt expected profit maximization as the manager's objective, in spite of the widespread belief that this is not a common motivation in centrally planned economies. We do so for several reasons. (1) Agreement on the inappropriateness of expected profit maximization in central planning is not unanimous and some past history suggests that it is, in fact, appropriate; thus after 1968 in Hungary it became part of the economic reform that enterprises should have control over their profits and that profit after taxes should act as an incentive (Zwass (1984)). (2) At a more practical level, any theory of the firm requires a maximand and some constraints. No determinate input or output decisions can be derived from noting only that the enterprise must maximize output and that all losses will be reimbursed by an outside agency. Profit maximization thus provides a natural first assumption under which we can evaluate the effects of a soft budget constraint. This has the added virtue of shedding light on how a soft-budget constraint might work in free market firms.

In addition to the specification of the objective function, a formal analysis requires a precise characterization of the soft-budget constraint. The literature, at best, provides only a rough guide as to what happens when enterprises incur losses. Our strategy is to view the enterprise in such instances as being able to use specialized bailout labor to "whine" in the appropriate bureaucratic corridors, but to assume that the payoff to this activity is subject to uncertainty. Moreover, since bailout labor is not costless, enterprises will not in general behave as if all losses will be forgiven. Of course, by varying the cost of bailout labor, the marginal produc-
tivity of whining, and the bailout uncertainty one can make the relevant constraint as "soft" as desired.

3. Models of the Firm With Bailout

We assume that we are dealing with an enterprise that has a strictly concave production function \( f(x_1, x_2) \). The argument \( x_1 \) is a composite input and \( x_2 \) is managerial labor.\(^3\) The firm is a price-taker in input and output markets; the wages of the two types of input are \( w_1 \) and \( w_2 \) and the output price is normalized to unity. Output is stochastic; the actual output that materializes is \( f(x_1, x_2)e^u \), where \( u \) is normally distributed with mean zero and variance \( \sigma^2 \). We refer to these assumptions as Assumption 1.

We denote the firm's operating profit by \( \Pi \). As indicated before, our models incorporate the notion of a "bailout" by the state in the eventuality that operating profits are negative; the bailouts are neither a fixed amount, nor random manna from heaven but are a positive function of the amount employed of a specialized type of managerial labor, \( x_3 \), which does not appear in the production function but is particularly well suited to whining in the corridors of government departments. As Kornai (1983, p.82) says:

The firm may also react [to changed circumstances] in a different way. It can attempt to influence the prices at which it buys and sells as well as financial variables (taxes, state subsidies, loans, etc.). In the first case, it is reacting in the realm of real variables, in the second in the realm of regulations. In the first case, it is acting in the plant in the second case in the ministry, in the tax office, in the offices of the bank.\(^4\)

Accordingly, we introduce

\textbf{Assumption 2.} If operating profit \( \Pi \) is negative, the firm receives a subsidy equal to \( h(x_3v)\Pi^{1/\theta} \), where \( 0 < h \leq 1, h(0) = 0, h' > 0, x_3 \) is the amount of specialized managerial labor devoted to bailout-seeking, and

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\(^3\)Most of the following arguments would be little changed if we suppressed the input \( x_1 \). We retain it for its slightly greater generality.

\(^4\)Our translation. Italics in the original.
v is a nonnegative random variable uncorrelated with u, with density function $g(v)$.

The particular manner in which randomness is introduced into bailout seeking corresponds to a situation in which randomness affects the efficiency of $x_3$-type labor; thus in a good state of nature, i.e., for large values of $v$, officials will be more receptive to bailout-seeking than in a bad state of nature. It is reasonable, though it turns out to be not necessary, to assume that the marginal bailout declines, i.e., that $h'' < 0$; it may also be useful at times to characterize the marginal bailout productivity as in

Assumption 3. (Inada condition): $\partial h / \partial x_3 \rightarrow 0$ as $x_3 \rightarrow 0$.

The timing of events is as follows. The firm employs certain quantities of $x_1$ and $x_2$ to produce an output. After the output materializes, i.e., after the production uncertainty is resolved, the firm can compute whether its operating profit is positive or negative. In the former case, no $x_3$ is needed, whereas in the latter, bailout seeking activity must be undertaken. In principle, therefore, the firm has a choice of committing itself to a certain amount of $x_3$ at the beginning (the amount that it would be good to have "on the average") or of waiting to see whether $x_3$ is needed after the production uncertainty is resolved. This distinction leads to two models which we call the "myopic" model and the "two-stage" model respectively. While the absence of precommitment may suggest that the two-stage model is preferable, the myopic model may well be more realistic.

A Model of Myopic Behavior. The operating profit of the firm may now be defined as

$$\bar{\pi} = f(x_1, x_2)e^u - w_1x_1 - w_2x_2 - w_3x_3 \quad (3.1)$$
The actual profit of the firm, \( \pi \), is given by

\[
\pi = \begin{cases} 
\pi & \text{if } \frac{\pi}{\pi} \geq 0 \\
\frac{\pi}{(1-h(x_3v))} & \text{if } \frac{\pi}{\pi} < 0
\end{cases}
\] (3.2)

which is in the spirit of Kornai and Weibull (1983).

To simplify notation, we define \( C = w_1x_1 + w_2x_2 + w_3x_3 \), and if it causes no ambiguity, we abbreviate \( f(x_1,x_2) \) by \( f \). We also note that the definition of operating profit includes as a negative term the cost of the specialized bailout-seeking labor.

We now assume that the firm maximizes \( E(\pi) \). Now \( \frac{\pi}{\pi} \geq 0 \) if \( fe^u - C \geq 0 \) or if

\[
u \geq \log(C/f) = \nu_0
\] (3.3)

When desired for clarity, we denote the dependence of \( \pi \) on \( u \) by \( \pi(u) \); we also use \( n(\mu, \sigma^2) \) to denote the normal density with mean \( \mu \) and variance \( \sigma^2 \). We then have

\[
E(\pi) = \Pr\{u \geq \nu_0\} \int_{\nu_0}^{\infty} \pi(u)n(0, \sigma^2|u\geq\nu_0)du + \\
\Pr\{u < \nu_0\} \int_{-\infty}^{0} \pi(u)n(0, \sigma^2|u<\nu_0)\int_{0}^{(1-h(x_3v))g(v)dv}du = \\
\int_{\nu_0}^{\infty} \pi(u)n(0, \sigma^2)du + \int_{\nu_0}^{\nu_0} \pi(u)\nu_3n(0, \sigma^2)du
\] (3.4)

Alternative assumptions worth exploring in the context of the socialist enterprise are (1) that the manager has to meet an output target and is penalized for deviations from it, and (2) that some inputs are rationed. We plan to explore both of these in future work.
from (3.5) we note that

\[ \psi'(x_3) = -\int_0^\infty \frac{h(x_3v)}{x_3} v g(v) dv \]

so that if Assumption 3 holds, \( \lim_{x_3 \to 0} \psi'(x_3) = -\infty \).

It is easy to verify that (3.4) may also be written as

\[ E(\pi) = f e^{\sigma^2/2} - C + (\psi(x_3)-1) \left[ f e^{\sigma^2/2} \left( \frac{u_o-\sigma^2}{\sigma} \right) - C + \left( \frac{u_o}{\sigma} \right) \right] \]

where \( \Phi(z) = \left( \frac{1}{\sqrt{2\pi}} \right) \int \exp(-x^2/2) dx \).

The interesting questions concern the comparison of the solution values obtained by maximizing (3.6) with what we designate as the (standard) competitive solution; i.e., the solution when no bailout mechanism exists.

The competitive solution \( x^0 = (x_1^0, x_2^0, x_3^0) \) is obviously given by \( x_3^0 = 0 \) and \( x_1^0, x_2^0 \) being the solutions of \( \partial E(\pi)/\partial x_i = 0 \), \( i = 1, 2 \). We restrict our attention to homogeneous production functions of degree \( s \), \( 0 < s < 1 \), and thus at \( x^0 \) we have \( C = s f e^{\sigma^2/2} \). Now denote \( \Phi\left( \frac{u_o-\sigma^2}{\sigma} \right) \) by \( \Phi_2 \), \( \Phi\left( \frac{u_o}{\sigma} \right) \) by \( \Phi_1 \) and the corresponding densities evaluated at \( (u_o-\sigma^2)/\sigma \) and \( u_o/\sigma \) by \( \Phi_2, \Phi_1 \) respectively. We have the following Lemmas.

**Lemma 1:** Evaluated at \( x^0 \), \( s\Phi_1 = \Phi_2 \).

**Proof:** \( u_o = \log(C/f) = \log(s) + \sigma^2/2 \). Substituting in the \( \Phi_1 \) and \( \Phi_2 \) functions and taking logarithms yields equality for the two sides.

**Lemma 2:** Evaluated at \( x^0 \), \( \Phi_2 - s\Phi_1 < 0 \).
Proof: Define \( \lambda(\log(s)) = \Phi_2 - e^{\log(s)} + 1 \). First note that for \( s=1 \), \( \lambda(0) < 0 \). Also, \( \lambda'(\log(s)) = \Phi_2 - s\Phi_1 - e^{\log(s)} + 1 \) which is negative by Lemma 1 for all \( s>0 \). Further, \( \lim_{s \to 0} \lambda(\log(s)) = 0 \) and \( \lambda \) is continuous for all \( s>0 \). It follows that \( \lambda \) cannot become \( \geq 0 \) for any \( s>0 \).

We now show that the solution values \( \hat{x} = (\hat{x}_1, \hat{x}_2, \hat{x}_3) \) that maximize (3.6) exceed the competitive levels given by \( x^0 \). Thus the myopic response to bailouts is to increase the demand for all inputs. (The practical question of how sensitive the solution for input demand is to the parameters of the problem is taken up in the next section.) We formalize the basic result as the following

**Theorem 1:** \( \hat{x} > x^0 \).

**Proof.** We show that the partial derivatives of (3.6) with respect to \( x_i \), \( i = 1, 2, 3 \), evaluated at \( x^0 \) are positive. We first consider \( x_3 \).

From the definition of \( \psi(x_3) \) and Assumption 2 we also have that \( \psi'(x_3) < 0 \); moreover \( \psi(0) = 1 \).

Differentiating (3.6) with respect to \( x_3 \), we obtain

\[
\frac{\partial E(\pi)}{\partial x_3} = -w_3 + (\Phi_2 - s_1 + 1) \Phi_2^{2/2} \frac{\partial \psi}{\partial x_3} + [\text{terms containing } 1-\psi] \tag{3.7}
\]

The terms containing \( 1-\psi \) vanish at the competitive solution. The second term is positive (by Lemma 2 and since \( \psi/x_3 < 0 \)). Furthermore, by Assumption 3, this term is arbitrarily large in the neighborhood of \( x_3 = 0 \), so the enterprise will always use some amount of \( x_3 \).

We now examine the partial derivatives of (3.6) with respect to \( x_1, x_2 \) and evaluated at the competitive solution. Since \( E(\pi) = \Phi_2^{2/2} - C \), we have

\[
\frac{\partial E(\pi)}{\partial x_1} = \frac{\partial E(\pi)}{\partial x_1} + (\psi(x_3) - 1) \left[ \text{derivative of bracketed term in (3.6)} \right] = 0 \tag{3.8}
\]

since \( \partial E(\pi)/\partial x_1 = 0 \) and \( \psi(x_3) = \psi(0) = 1 \) at \( x^0 \). Thus, the "first order" effect on expected profit from increasing \( x_1 \) or \( x_2 \) appears to be nil. Since
\(\frac{\partial^2 E(\pi)}{\partial x_i \partial x_3}\) is already known to be positive, we also need \(\frac{\partial^2 E(\pi)}{\partial x_i \partial x_3}, i = 1,2\) which derived in the Appendix to be

\[
\frac{\partial^2 E(\pi)}{\partial x_i \partial x_3} = -w_i (\psi_1 - \psi_2) \frac{\partial \psi}{\partial x_3} \tag{3.9}
\]

This is positive, since \(\psi_1 - \psi_2 > 0\) and \(\frac{\partial \psi}{\partial x_3} < 0\). It follows from Lemma 3 in the Appendix that \(E(\pi)\) increases faster along a vector \((\Delta x_1, \Delta x_2, \Delta x_3)\) with all positive components than along a vector which has the first two components equal to zero. Intuitively, at the competitive solution the enterprise only has the incentive to increase \(x_3\); but as soon as \(x_3\) has increased, the derivatives with respect to \(x_1\) and \(x_2\) also turn positive and the amounts of both inputs increase.

Although this is only a "local" argument, it provides a foundation for the view that a soft-budget constraint\(^6\) leads to an increase in the demand for inputs. The softness of the budget constraint is perhaps best measured by the marginal productivity of the bailout-seeking activity; hence it will be interesting to see how sensitive input demands are to this.

**Remark 1.** An example of an \(h\) function satisfying the conditions of the theorem is provided by \(h(x_3v) = 1 - e^{-ax_3v}\), where \(0 < a < 1\).\(^7\) If \(v\) has exponential density \(g(v) = be^{-bv}\), we have \(\psi(x_3) = b/(b+ax_3^c)\) and \(\frac{\partial \psi}{\partial x_3}\) becomes an arbitrarily small number as \(x_3 \to 0\) as expected, since \(h\) satisfies Assumption 3.

**Remark 2.** If Assumption 3 does not hold, the enterprise will not wish to hire any of the specialized management labor in certain circumstances when we start from the competitive solution. That is, in (3.7) we may have

\(^6\)Note that we are not distinguishing here between Gomulka’s (1985) notions of soft and flexible budget constraints.

\(^7\)This is consistent with having a multiplicative error represent the effective amount of \(x_3\) available, since it can be rewritten as \(h(x_3v) = 1 - \exp(-a(x_3v^1/c)^c)\).
\( aE/\partial x_3 < 0 \). The actual outcome depends on the nature of the functions \( h(x_3v) \), \( f \) and \( g(v) \). If, for example, \( h(x_3v) = 1 - e^{-ax_3v} \), and \( v \) again has the exponential density, then \( \psi(x_3) = b/(ax_3+b) \), and \( \partial \psi/\partial x_3 \) evaluated at \( x_3 = 0 \) is \(-a/b\). By examining (3.7) we can tell immediately that the likelihood of a positive \( aE(\pi)/\partial x_3 \) is greater under any of the following conditions. (1) If \( a \) is large, i.e., if the marginal bailout-productivity of \( x_3 \)-type labor is large; (2) if \( b \) is small, which is equivalent to saying that the probability of favorable states of nature (\( v > 1 \)) is large; (3) if \( \sigma^2 \) is large, i.e., if the variance of the production uncertainty is large; (4) if \( s \) is large, since \( \partial(\mu^2-s^1)/\partial s < 0 \).

A Two-Stage Model. The previous, myopic, model involved the enterprise to commit itself to \( x_1, x_2 \) and \( x_3 \). An alternative procedure is as follows: the enterprise hires certain quantities of \( x_1 \) and \( x_2 \) and produces an output; after the resolution of this output uncertainty, the enterprise decides how much \( x_3 \) to hire. The optimal solution of this problem is then calculated, as usual, in reverse order. We first maximize expected profit in the second stage and use that solution to determine the optimal quantities of \( x_1 \) and \( x_2 \).

We now define operating profit as

\[
\pi_1 = f(x_1, x_2)e^u - w_1x_1 - w_2x_2 .
\]

(3.10)

If \( \pi < 0 \), the enterprise wishes to hire some \( x_3 \) in order to obtain subsidies. In this second stage we therefore maximize the expectation of

\[
\pi_2 = \pi_1(1-h(x_3v)) - w_3x_3
\]

(3.11)
i.e., minimize the net loss after subsidy. At this stage only \( v \) is random and hence

\[
E(\pi_2) = \pi_1\psi(x_3) - w_3x_3 = A(x_3)
\]

(3.12)

where \( \psi(x_3) \) is, as before, given by (3.5). To maximize, we set \( A'(x_3) = 0; \)
this determines $x_3^*$ such that $A(x_3^*) = \max_{x_3} A(x_3)$. As before, the Inada condition (Assumption 3) ensures that $x_3^* > 0$, for differentiating (3.12),

$$\pi_1 \psi'(x_3) = w_3 \quad (3.13)$$

We see that (3.13) cannot be satisfied by $x_3 = 0$ if $\psi'(0) = -\infty$. The first stage optimization is then based on noting that

$$\pi = \begin{cases} 
\pi_1 & \text{if } \pi_1 \geq 0 \\
A(x_3^*) & \text{if } \pi_1 < 0 
\end{cases} \quad (3.14)$$

Hence

$$E(\pi) = \int_{u_0}^{\infty} \pi_1(u)n(0,\sigma^2)du + \int_{-\infty}^{u_0} A(x_3^*)n(0,\sigma^2)du \quad (3.15)$$

where $u_0$ is determined by (3.3) as before, except that $C$ is defined here as $w_1x_1 + w_2x_2$. Comparing (3.15) with (3.4) and (3.6) shows that it can also be written as

$$E(\pi) = \int_{-\infty}^{u_0} \frac{\sigma^2}{2} (1-\Phi_2) - C(1-\Phi_1) + \int A(x_3^*)n(0,\sigma^2)du \quad (3.16)$$

We now show that the possibility of bailouts unambiguously increases $x_1$ and $x_2$ over the competitive levels. If $x_1^*, x_2^*$ denotes the input levels that maximize (3.16), we have formally

**Theorem 2.** $(x_1^*, x_2^*) > (x_1^0, x_2^0)$.

**Proof.** To show this we require the partial derivatives of (3.16). We first note from (3.13) that

$$x_3 = \psi' - 1 \left( \frac{w_3}{\pi_1} \right) \quad (3.17)$$

which implies by Assumption 3 that as $\pi_1$ approaches zero from the left, $x_3 \to 0$.

When we differentiate (3.16), the derivative of the integral would normally yield two terms by Leibnitz' rule. However, one of these is zero, since
at $u_0$, which is the upper limit of the integral in (3.16), $A(x_3^*) = 0$. This latter follows from the observations that $\pi_1(u_0) = 0$ (by definition) and that at $u_0$ we have $x_3^* = 0$ (by (3.12)). We shall require for $i = 1, 2$

$$\frac{\partial A(x_3)}{\partial x_i} = \pi_1^* \frac{\partial x_3}{\partial x_i} + \pi_1^* \psi(x_3^*) - \psi_3 \frac{\partial x_3}{\partial x_i}$$

$$= \frac{\partial \pi_1}{\partial x_i} \psi(x_3^*) = (f_i e^u - \psi_i) \psi(x_3^*)$$

by (3.13). The derivatives of the integral are then

$$\frac{\partial}{\partial x_i} \left[ \int_{-\infty}^{u_0} A(x_3)n(0, \sigma^2) du \right] = \int_{-\infty}^{u_0} (f_i e^u - \psi_i) \psi(x_3^*) n(0, \sigma^2) du$$

$$= \psi_i \int_{-\infty}^{u_0} (e^{-\sigma^2/2} - 1) \psi(x_3^*) n(0, \sigma^2) du$$

(3.18)

by using the properties of the competitive solutions. The derivatives of the first two terms in (3.16) are

$$\psi_1(1+\psi_2) - \frac{\psi_i e^{\sigma^2/2}}{s} \frac{\psi_i E(\pi_1)}{\sigma C e^{\sigma^2/2}} - \psi_1(1+\psi_1)$$

$$+ C \phi_1 \frac{\psi_i E(\pi_1)}{\sigma C e^{\sigma^2/2}}$$

$$\psi_i(\phi_1 - \phi_2) - \frac{\psi_i E(\pi_1)}{\sigma C e^{\sigma^2/2}} \left( \frac{\phi_2}{s} - \phi_1 \right) = \psi_i(1+\phi_2)$$

(3.19)

by Lemma 1. Combining (3.18) and (3.19) gives

$$\frac{\partial E}{\partial x_i} = \psi_i \left[ \psi_1 - \psi_2 + \int_{-\infty}^{u_0} (e^{u_{-\sigma^2/2}} - 1) \psi(x_3^*) n(0, \sigma^2) du \right] i = 1, 2$$

(3.20)
In the integral \( u \leq u_0 = \log s + \sigma^2/2 \), with \( s < 1 \); hence the parenthesis is negative. Also \( \psi(x_3^*) \leq 1 \). Thus, the integral in (3.20) is greater than or equal to

\[
\int_{-\infty}^{u_0} (e^{u-\sigma^2/2} - 1)n(0,\sigma^2)du = \ast_2 = 1
\]

It follows that \( \frac{\partial E}{\partial x_1} > 0 \). Thus, the two-stage process unambiguously increases the demand for inputs compared to the competitive level.

4. Some Numerical Results

In Section 3 we obtained general qualitative results concerning bailout-seeking behavior and its consequences. However, the theorems of the previous section do not, by themselves, allow us to characterize the quantitative importance of the effects that we noted. This can be accomplished by numerical experiments in which specific functional forms and parameter values are posited. While the results of such experiments are only illustrative, they can nevertheless shed substantial further light on the underlying relations; moreover, they provide a tool with which additional scenarios can also be examined.

The Basic Setup. It is, of course, a requirement for numerical experiments that specific production and bailout functions be assumed. We take these to be

\[
f(x_1, x_2) = x_1^\alpha x_2^\beta e^u \tag{4.1}
\]

where \( u \) is normally distributed with mean zero and variance \( \sigma^2 \) and

\[
h(x_3v) = d - \exp(-ax_3^Cv) \tag{4.2}
\]

where \( v \) is distributed exponentially, with \( g(v) = be^{-bv} \). All parameters

\[\text{We have also performed some experiments in which } v \text{ was assumed to be Gamma-distributed. For the sake of brevity, we omit these results.}\]
\(\alpha, \beta, a, b, c, d, w_3, \sigma^2\) are positive and \(0 < c < 1\) and \(0 < d < 1\). The particular form of (4.2) allows bailout to be less than complete, even if \(x_3\) is arbitrarily large, whenever \(d < 1\). It follows from (3.5) that with this bailout function

\[
\psi(x_3) = \frac{b}{ax_3^2 + b} + 1 - d.
\]

We vary scenarios by choosing alternative sets of specific values for the parameters. For each scenario we obtain the solution for the myopic model by maximizing (3.6) and for the two-stage model by maximizing (3.15) or the equivalent (3.16). This requires numerical optimization of the highly non-linear functions.\(^9\)

In order to keep the scope of experiments within bounds, we did not vary all the parameters. Throughout we assume \(w_1 = w_2 = 0.4\) and \(a = \beta\). This in effect reduces the production function to be a function of only one factor.

We contrast the various bailout solutions with two extreme cases. The first of these is the competitive solution; i.e., the solution in which no bailout is permitted and which maximizes \(E(\pi) = f e^{\sigma^2/2} - w_1 x_1 - w_2 x_2\). The second is the complete bailout case in which complete loss-offset is provided at no cost to the enterprise; this involves maximizing \(E(\pi) = f e^{\sigma^2/2} (1 - \pi) - C(1 - \pi),\) which is obtained from (3.6) by setting \(\psi(x_3) = 0\). These two cases are, so to speak, the numeraire cases that "calibrate" the intermediate (in-

\(^9\)We employed the optimization package GQOPT, available from the authors. Computations were performed on an IBM 3081-K. It may be noted that the procedure is particularly complicated in the case of (3.16) because no closed-form solution for \(x_3^\beta\) is available. Clearly, evaluation of the last term of (3.16) requires numerical integration with respect to \(u\), beyond the implicit numerical integration involved in \(\pi_1\) and \(\pi_2\). But as we integrate along the u-axis, each value of \(u\) determines a new value of \(\pi_1\), which from \(A'(x_3) = 0\) determines a new, numerically obtained value of \(x_3^{\beta}\). Thus the overall optimization requires for each evaluation of the function to be maximized a numerical integral in which, for each integrand value computed, a non-linear equation must be solved numerically.
complete) bailout scenarios. The solution values of the extreme cases are affected only by the parameters $\alpha$ and $\sigma^2$; the incomplete bailout cases are affected by the remaining parameters as well.

Results for the Numeraire Cases: Table 1 displays $x_1, x_3, \Pr(\bar{\pi} < 0)$, $E(\pi)$ and $E(\bar{\pi})$. Note that $\bar{\pi} = fe^u - \omega_1x_1 - \omega_2x_2$ in these cases and that in the competitive case $E(\pi)$ and $E(\bar{\pi})$ are identical by definition. The table reveals that for given $\alpha$ the solution is somewhat sensitive to variations in $\sigma^2$ in the competitive case ($x_1$ increasing 45% from line 1 to line 5) and very sensitive in the complete bailout case ($x_1$ increasing nearly 28-fold from line 2 to line 6). Several of the complete bailout scenarios have negative expected $\bar{\pi}$; in these cases there is also a very high probability that $\bar{\pi}$ will be negative. The intuition here is that if the manager is assured of having any possible losses wiped out, he will be acting rationally if he substantially increases his input demand, thus causing an operating loss.
It should be noted that the values of $\sigma^2$ employed in these cases are very large in the sense that a 95% confidence interval for $e^u$ is $(0.54, 1.86)$ for $\sigma^2 = 0.1$ and $(0.73, 1.36)$ for $\sigma^2 = 0.025$. Since the uncertainty due to $u$ is production uncertainty, realistic figures for $\sigma^2$ are likely to be smaller than the ones we have employed. Consequently, other things being equal, the likely effect of even complete bailout will be less than indicated in the first six rows of Table 1. However, as Table 1 also indicates, with $\sigma^2$ fixed increases in $\alpha$ serve to widen the scope for bailouts. Indeed, large values of $\alpha$ provide a scope for bailouts, even if $\sigma^2$ is quite small.\(^{10}\)

Our primary interest, of course, is the sensitivity of the incomplete bailout solutions to variations in the underlying parameters. These solutions for $x_1$ lie between the competitive and complete bailout solutions; hence, if these solutions are quite close there is little scope for studying the sensitivity of incomplete bailout solutions. Without prejudging the empirical issues, for the remaining experiments we fixed $\alpha = 0.4$ and $\sigma^2 = 0.1$, values large enough to produce some interesting variations. Thus, in what follows, the comparable competitive and complete bailout solutions are given in rows 3 and 4 of Table 1.

Results for the Myopic Model. In the experiments with this model, the base case is given by the following parameter values: $a = 5.0$, $b = 0.5$, $c = 0.5$, $d = 1.0$ and $w_3 = 0.4$. All of these parameters were subjected to variation, one at a time. The basic results are in Table 2. The first row displays the base case; the remaining rows show in column 1 the way in which a particular case differs from the base case. All cases behave sensibly and show that the more easily bailouts are obtained, the more the scale of the enterprise will be pushed beyond the competitive level. An increase in $\sigma^2$\(^{10}\) For example, with $\sigma^2 = 0.003$ and $\alpha = 0.47$ the complete bailout solution for $x_1$ is 44% larger than the competitive solution.
represents an increase in the productivity of bailout-seeking behavior and as a result the demand for $x_1$ increases, expected profit increases, and $E(\bar{\pi})$ decreases. The demand for $x_3$ is particularly interesting as it first increases and then declines as $a$ rises. The probability that bailout will be required, which is the same as the probability that $\bar{\pi} < 0$, does, however, monotonically increase with $a$.

An increase in $b$ has the opposite affects from $a$ which is to be expected, since for our choice of $g(\ )$ and $h(\ )$ functions, $\psi$ contains $a$ and $b$ only as the ratio $a/b$. As $d$ increases, bailouts can become more complete for large $x_3$; this has precisely the same affects as an increase in $a$. An increase in

<table>
<thead>
<tr>
<th>Case</th>
<th>$x_1$</th>
<th>$x_3$</th>
<th>$Pr(\pi &lt; 0)$</th>
<th>$E(\pi)$</th>
<th>$E(\bar{\pi})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>2.344</td>
<td>0.064</td>
<td>0.451</td>
<td>0.300</td>
<td>0.203</td>
</tr>
<tr>
<td>$a=1.0$</td>
<td>1.419</td>
<td>0.013</td>
<td>0.319</td>
<td>0.262</td>
<td>0.256</td>
</tr>
<tr>
<td>$a=50.0$</td>
<td>4.551</td>
<td>0.059</td>
<td>0.608</td>
<td>0.369</td>
<td>-0.107</td>
</tr>
<tr>
<td>$b=0.1$</td>
<td>3.948</td>
<td>0.072</td>
<td>0.576</td>
<td>0.353</td>
<td>-0.005</td>
</tr>
<tr>
<td>$b=0.75$</td>
<td>2.004</td>
<td>0.051</td>
<td>0.411</td>
<td>0.287</td>
<td>0.230</td>
</tr>
<tr>
<td>$d=0.5$</td>
<td>1.333</td>
<td>0.018</td>
<td>0.308</td>
<td>0.254</td>
<td>0.257</td>
</tr>
<tr>
<td>$d=0.75$</td>
<td>1.620</td>
<td>0.028</td>
<td>0.355</td>
<td>0.271</td>
<td>0.250</td>
</tr>
<tr>
<td>$c=0.25$</td>
<td>2.738</td>
<td>0.034</td>
<td>0.481</td>
<td>0.326</td>
<td>0.163</td>
</tr>
<tr>
<td>$c=0.75$</td>
<td>2.032</td>
<td>0.082</td>
<td>0.423</td>
<td>0.278</td>
<td>0.228</td>
</tr>
<tr>
<td>$w_3=0.2$</td>
<td>2.674</td>
<td>0.144</td>
<td>0.484</td>
<td>0.312</td>
<td>0.170</td>
</tr>
<tr>
<td>$w_3=1.0$</td>
<td>1.964</td>
<td>0.020</td>
<td>0.405</td>
<td>0.285</td>
<td>0.233</td>
</tr>
</tbody>
</table>

c reduces $x_1$ and increases $x_3$; $E(\pi)$ declines but $E(\bar{\pi})$ increases. The reason is that an increase in $c$ increases $\psi$ if $x_3 < 1$ (which it is in all cases here); hence, expected profit is adversely affected, leading to an increase in the employment of bailout-seeking labor. This is then compensated by a reduction in $x_1$. Finally, an increase in $w_3$ has the predictable effect of reducing

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both inputs, \( \Pr[\bar{r}<0] \) and \( E(\bar{r}) \) but increasing \( E(\bar{r}) \).

Table 3 shows the arc-elasticities of the various solution values with respect to the parameter values that produced them, in all cases relative to the base.

<table>
<thead>
<tr>
<th>Case</th>
<th>( x_1 )</th>
<th>( x_3 )</th>
<th>( \Pr[\bar{r}&lt;0] )</th>
<th>( E(\bar{r}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a=1.0 )</td>
<td>0.369</td>
<td>0.021</td>
<td>0.256</td>
<td>0.100</td>
</tr>
<tr>
<td>( a=50.0 )</td>
<td>0.391</td>
<td>-0.001</td>
<td>0.181</td>
<td>0.126</td>
</tr>
<tr>
<td>( b=0.1 )</td>
<td>-0.383</td>
<td>-0.002</td>
<td>-0.183</td>
<td>-0.122</td>
</tr>
<tr>
<td>( b=0.75 )</td>
<td>-0.391</td>
<td>-0.015</td>
<td>-0.233</td>
<td>-0.114</td>
</tr>
<tr>
<td>( d=0.5 )</td>
<td>0.487</td>
<td>0.018</td>
<td>0.356</td>
<td>0.164</td>
</tr>
<tr>
<td>( d=0.75 )</td>
<td>1.278</td>
<td>0.063</td>
<td>0.835</td>
<td>0.357</td>
</tr>
<tr>
<td>( c=0.25 )</td>
<td>-0.233</td>
<td>0.018</td>
<td>-0.096</td>
<td>-0.124</td>
</tr>
<tr>
<td>( c=0.75 )</td>
<td>-0.356</td>
<td>0.021</td>
<td>-0.158</td>
<td>-0.191</td>
</tr>
<tr>
<td>( w_3=0.2 )</td>
<td>-0.197</td>
<td>-0.048</td>
<td>-0.106</td>
<td>-0.060</td>
</tr>
<tr>
<td>( w_3=1.0 )</td>
<td>-0.206</td>
<td>0.024</td>
<td>-0.123</td>
<td>-0.060</td>
</tr>
</tbody>
</table>

It is interesting to note that most of the elasticities are very small and only one exceeds unity.

Results for the Two-Stage Model. Similar calculations were performed for the two-stage model.\(^{11}\) The results were extremely similar to those of the myopic model. For this reason we display only the arc elasticities in Table 4. One potentially puzzling phenomenon emerged from the comparison of the two models. One might have guessed that since the two-stage model avoids the

\(^{11}\)The details of the computations themselves were, of course, different. In particular, since no \( x_3 \) is employed ex ante, the model solves for \( x_1 \) and \( x_2 \) only. While these imply a value for \( \Pr[\bar{r}<0] \) and an expected value for \( x_3 \), analytic expressions for these are not readily available. However, given \( x_1 \) and \( x_2 \), a \( \bar{r} \) can be computed if one is given a realization of the disturbance \( u \). By generating drawings from the normal distribution 10,000 times, we obtained a Monte Carlo estimate of \( \Pr[\bar{r}<0] \) and of the expected value of \( x_3 \).
Table 4. Arc Elasticities for Two-Stage Model

<table>
<thead>
<tr>
<th>Case</th>
<th>$x_1$</th>
<th>$x_3$</th>
<th>$\text{Pr}(\pi &lt; 0)$</th>
<th>$E(\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = 1.0$</td>
<td>0.303</td>
<td>0.006</td>
<td>0.186</td>
<td>0.098</td>
</tr>
<tr>
<td>$a = 50.0$</td>
<td>0.328</td>
<td>-0.003</td>
<td>0.165</td>
<td>0.099</td>
</tr>
<tr>
<td>$b = 0.1$</td>
<td>-0.306</td>
<td>0.002</td>
<td>-0.168</td>
<td>-0.094</td>
</tr>
<tr>
<td>$b = 0.75$</td>
<td>-0.283</td>
<td>-0.001</td>
<td>-0.127</td>
<td>-0.091</td>
</tr>
<tr>
<td>$d = 0.5$</td>
<td>0.471</td>
<td>0.010</td>
<td>0.343</td>
<td>0.166</td>
</tr>
<tr>
<td>$d = 0.75$</td>
<td>1.098</td>
<td>0.024</td>
<td>0.633</td>
<td>0.339</td>
</tr>
<tr>
<td>$c = 0.25$</td>
<td>-0.210</td>
<td>0.010</td>
<td>-0.154</td>
<td>-0.086</td>
</tr>
<tr>
<td>$c = 0.75$</td>
<td>-0.253</td>
<td>0.017</td>
<td>-0.120</td>
<td>-0.114</td>
</tr>
<tr>
<td>$w_3 = 0.2$</td>
<td>-0.145</td>
<td>-0.022</td>
<td>-0.070</td>
<td>-0.046</td>
</tr>
<tr>
<td>$w_3 = 1.0$</td>
<td>-0.149</td>
<td>-0.012</td>
<td>-0.085</td>
<td>-0.048</td>
</tr>
</tbody>
</table>

precommitment to $x_3$, and in fact uses $x_3$ only if actually necessary, the expected profit in the two-stage model would always exceed the expected profit in the myopic model for the corresponding case. However, this was not uniformly the case. The reason is to be found in a subtle difference between the two models. In the myopic model, bailouts are obtainable for operating losses that arise because bailout seeking labor was hired -- something that is not the case in the two-stage model. It turns out that if the myopic model is amended so that the portion of any loss accounted for by the $w_3x_3$ term is not reimbursable by bailouts, then the expected profit in the two-stage model is always greater than in the myopic model.

5. Conclusions

We have formulated two models of an enterprise that can obtain bailouts from the state when its operating profit is negative. Our models are motivated by the notion of the soft budget constraint that is encountered frequently in socialist economies. The essential features of our models are (1) expected profit maximization, (2) the availability of a specialized labor
the output of which is subsidies, (3) uncertainty in production, (4) uncertainty in the effectiveness with which the specialized labor secures bailouts. The two models differ from one another in the degree of precommitment; in the myopic model the specialized labor is hired before any randomness is resolved, whereas in the two-stage model it is hired only if the operating profit is observed to be negative.

Two theorems are proved that show that the employment of inputs in both models is greater than in the competitive case (i.e., the simple case in which bailouts are not available). This underscores Kornai’s arguments that the soft budget constraint leads to increased demands for inputs. Computation with numerical examples shows, however, that if production uncertainty is small, and if the bailout is not complete, this effect is greatly attenuated. In general, variation in the parameters of the problem changes solution values in the a priori expected manner but the elasticities of the solution values with respect to the parameters are also predominantly small.

Several realistic extensions of these models will be addressed in future work. We briefly mention two whose relevance for the socialist enterprise is evident. The first of these is the case in which the manager is given an output target \( \bar{q} \). With production uncertainty present, it is not quite reasonable to model this as

\[
\text{maximize } E(\pi) \\
\text{Subject to } fe^u \geq \bar{q}
\]

for there is no action by the manager that will guarantee that the constraint is satisfied. It may be more reasonable to argue that the manager’s utility function is expected profit minus a penalty that is quadratic in the deviation of output from target.\(^{12}\) Using the myopic model as an example, \(^{12}\) Obviously the penalty may be symmetric or asymmetric for positive and negative deviations.
\[ \pi = \begin{cases} \bar{\pi} - \delta(f_{eu} - \bar{q})^2 & \text{if } \bar{\pi} \geq 0 \\ \bar{\pi} \left(1 - h(x_{3v})\right) - \delta(f_{eu} - \bar{q})^2 & \text{if } \bar{\pi} < 0 \end{cases} \]

It is easy to show that \( E(\pi) \) is the same as in (3.6) minus the term \( \delta[f_{2e}^2\sigma^2 - 2\bar{q}f_{e}\sigma^2/2 + \bar{q}^2] \).

A second realistic modification is to assume that one or more inputs are rationed. One might assume that, say, \( x_1 \) is rationed and that \( x_1 \) may not exceed \( \bar{x}_1 \). It seems reasonable also to assume that rationing is stochastic, i.e., that \( \bar{x}_1 \) has a probability density function \( \rho(\bar{x}_1) \) with nonnegative support. An expected profit function can again be formulated. Both of these features could also be combined in a single model. All of these cases promise to be more complicated with several new possibilities. A full analysis will be attempted in subsequent work.
Lemma 3. Let $G(x_1, x_2, x_3)$ be a twice differentiable function such that at $x^0 = (x_1^0, x_2^0, x_3^0)$ we have $G_1(x^0) = G_2(x^0) = G_3(x^0) > 0$ and also $G_{12}(x^0) > 0$, $G_{13}(x^0) > 0$. Then there exists a vector $\Delta x$ from $x^0$ with all three components positive, along which $G$ is increasing faster than along the vector $(0,0,\Delta x_3)$.

Proof. We are interested in the change of $G$ along the vector $\Delta x$ from $x^0$. We have

$$\frac{\partial G(x^0+\lambda \Delta x)}{\partial \lambda} = \nabla G(x^0+\lambda \Delta x) \Delta x$$

where $\nabla$ denotes the gradient. Also we have the Taylor Series expansion

$$\nabla G(x^0+\lambda \Delta x) = \nabla G(x^0) + \lambda^* H(x^0) \Delta x$$

where $0 < \lambda^* < \lambda$ and $H(x^0)$ is the matrix of second partial derivatives.

Thus

$$\frac{\partial G(x^0+\lambda \Delta x)}{\partial \lambda} = \nabla G(x^0) \Delta x + \lambda^* \Delta x^T H(x^0) \Delta x$$

By hypothesis, the first two components of $\nabla G(x^0)$ are zero and $H_{13}(x^0)$, $H_{23}(x^0)$ are positive. Denote the derivative $\frac{\partial G}{\partial \lambda}$ when $\Delta x_1 = \Delta x_2 = 0$ as $(\frac{\partial G}{\partial \lambda})_1$ and when $\Delta x_1 > 0$, $\Delta x_2 > 0$ as $(\frac{\partial G}{\partial \lambda})_2$; the value of $\Delta x_3$ being fixed at the same level in either case. Then

$$(\frac{\partial G}{\partial \lambda})_2 - (\frac{\partial G}{\partial \lambda})_1 = \lambda^* \left[ H_{11}(\Delta x_1)^2 + H_{22}(\Delta x_2)^2 ight.$$

$$
+ 2H_{12}(\Delta x_1)(\Delta x_2) + 2H_{13}(\Delta x_1)(\Delta x_3) + 2H_{23}(\Delta x_2)(\Delta x_3) \left.] \right]$$

It is clear that irrespective of the signs of $H_{11}$, $H_{22}$, $H_{12}$, this expression can be made to be positive by taking $\Delta x_1$, $\Delta x_2$ small enough, which establishes the Lemma.
Proof of (3.6). Rewrite $E(\pi)$ as

$$E(\pi) = E(\pi) \left[ 1 - \psi_1(1 - \psi(x_3)) \right] + (\psi_1 - \psi_2)fe^{\sigma^2/2}(1 - \psi(x_3))$$

where $\psi_1 = \psi(u_0/\sigma)$, $\psi_2 = \psi((u_0 - \sigma^2)/\sigma)$.

Differentiating yields directly

$$\frac{\partial E(\pi)}{\partial x_1} = \frac{\partial E(\pi)}{\partial x_1} \left[ 1 - \psi_1(1 - \psi(x_3)) \right] - E(\pi)(1 - \psi(x_3)) \frac{\partial \psi_1}{\partial x_1} +$$

$$\left( \frac{\partial \psi_1}{\partial x_1} - \frac{\partial \psi_2}{\partial x_1} \right)fe^{\sigma^2/2}(1 - \psi(x_3)) + (\psi_1 - \psi_2)fe^{\sigma^2/2}(1 - \psi(x_3)) i = 1, 2 \quad (A.1)$$

Proof of (3.9). Differentiating (A.1), we have

$$\frac{\partial^2 E(\pi)}{\partial x_1 \partial x_3} = \frac{\partial E(\pi)}{\partial x_1} \phi_1 \frac{\partial \psi}{\partial x_1} - fe^{\sigma^2/2} \left( \frac{\partial \psi_1}{\partial x_1} - \frac{\partial \psi_2}{\partial x_1} \right) \frac{\partial \psi}{\partial x_3} - fe^{\sigma^2/2} \left( \psi_1 - \psi_2 \right) \frac{\partial \psi}{\partial x_3} =$$

$$\frac{\partial \psi}{\partial x_3} \left[ \phi_1 \frac{\partial E(\pi)}{\partial x_1} \right] \left[ + fe^{\sigma^2/2} \frac{\partial \psi_1}{\partial x_1} - \frac{\partial \psi_2}{\partial x_1} \right] \frac{\partial \psi}{\partial x_3} \left[ \phi_1 \frac{\partial E(\pi)}{\partial x_1} \right] - \psi_1 \left( \psi_1 - \psi_2 \right) \frac{\partial \psi}{\partial x_3} \left[ \phi_1 \frac{\partial E(\pi)}{\partial x_1} \right]$$

$$\left( A.2 \right)$$

where we have used the facts that

$$\frac{\partial \psi_1}{\partial x_1} = \phi \frac{u_0}{\sigma} \frac{\partial u_0}{\partial x_1}, \quad \frac{\partial \psi_2}{\partial x_1} = \phi \frac{u_0 - \sigma^2}{\sigma} \frac{\partial u_0}{\partial x_1},$$

$$\frac{\partial u_0}{\partial x_1} = \frac{\partial}{\partial x_1} \log(C/f) = \frac{\psi E(\pi)}{Cfe^{\sigma^2/2}}$$

and that at the competitive solution $fe^{\sigma^2/2} = \psi_1$. (A.2) further simplifies to

$$\frac{\partial^2 E(\pi)}{\partial x_1 \partial x_3} \frac{\partial \psi}{\partial x_3} \left[ \phi_1 \frac{\partial E(\pi)}{\partial x_1} \right] \left( \psi_2 - \phi_1 \right) - \psi_1 \left( \psi_1 - \psi_2 \right) \frac{\partial \psi}{\partial x_3}$$

but by Lemma 1, the first term is zero. This proves (3.9).
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