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TITLE: THE PARALLEL MARKET IN CENTRALLY PLANNED ECONOMIES

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1. Introduction

The "parallel" or "second" economy has become an integral part of many centrally planned economies (CPEs). Contributions by Grossman (1977, 1983), Katsenelinboigen (1977) and Simes (1975) provide detailed accounts of the pervasiveness of this phenomenon in the Soviet Union. On the basis of the evidence provided by these authors, it is perhaps safe to assert that the second economy can no longer be regarded as an unimportant or inconsequential part of CPEs. Indeed, in order to understand the official or "first" economy better, we must study its links with the unofficial or second economy.

The present paper constructs a series of general equilibrium models to understand some of these links. My emphasis is on looking at different ways in which illegal economic activities can be incorporated into a planned economy's model rather than generating a specific set of results. The analysis in the paper leads to a better overall understanding of the links between the first and second economies and provides at least a preliminary framework for incorporating the parallel market into empirically oriented models of CPEs.

2. Structure of the Models

The paper employs two-sector general-equilibrium models used commonly in areas of international trade theory and public economics. These models as well as the modifications introduced in the present paper require a fair degree of abstraction. The ultimate objective is to capture the basic relationships in the economy in an internally consistent manner without losing tractability. A schematic flow chart of the economy as hypothesized in the paper is displayed in Figure I.

We begin by postulating a CPE which produces two goods using two inputs,
labor and capital. One good employs more labor per unit of capital than the other and is referred to as the labor intensive good. The other good is called the capital intensive good. Capital is owned by the state. All official allocation decisions are made by the state’s planning agency. The latter uses appropriate shadow prices as signals to implement its optimally chosen plan.

It is assumed that there are two types of agents in the economy, workers and bureaucrats. The latter have a strong preference for the capital intensive good relative to the former. Workers are employed by enterprises and are paid wages in return for labor services. Bureaucrats work directly for the state and receive salaries out of state revenues.\(^1\) In reality, a large part of the state’s revenues is spent on capital formation and defense. But in order to keep matters simple, I assume that all revenues are given to bureaucrats. We can, of course, think of the latter’s strong preference for the capital intensive good as representing the demand for defense and investment goods.

There are several themes in the existing literature concerning the emergence of the parallel market. The present paper formalizes two of these themes. First, I develop the idea that private firms are more efficient than the bureaucratically-run state enterprises. As Desai (1986) puts it, “(T)he Soviet enterprises...are plagued by the Leibenstein disease of X-inefficiency...” Second, I analyze the implications of parallel markets which emerge due to price controls and quantitative allocations employed extensively in CPEs.

An obvious question concerns the source of inputs for producers in the

\(^1\)The state’s revenues consist of profits (or “surplus”) earned on capital and penalties collected from those caught participating in the second economy.
second economy. As the total endowments of capital and labor in the economy are fixed, the second economy must obtain its inputs from the first economy. Thus, I assume that enterprises sell a part of their capital to second economy producers for a profit. Analogously, workers divert a part of their services to the second economy in the hope of getting a higher wage. There is a good deal of support for these assumptions in the empirical literature cited in the first paragraph of this summary.

A final notable feature of the analysis is the explicit consideration of the state's enforcement policy. The state monitors illegal producers. This fact means that the second economy producers face an uncertain environment. Taking the probability of detection and punishment (involving complete confiscation of goods) as given, private producers maximize expected profits. A separate section (Section 3) of the paper considers the case when the state monitors the resource diversion activity of enterprises.

As noted earlier, Figure I gives a schematic flow chart of the first and second economies as hypothesized in the paper.

3. Analysis and Conclusions

The analysis and conclusions of the paper can be understood most easily by assuming that the parallel market exists in only one sector. Extensions to the case when both goods are subject to illegal production are straightforward and can be found in the paper. In the following, it will be assumed that only the labor intensive sector is subject to illegal production.

Let us begin by considering the case when private firms emerge due to the fact that they can operate more efficiently than bureaucratically-run state enterprises. In order to model these firms, shortages which characterize most CPEs are unessential. Therefore, I begin with the simple framework in which the planning agency implements a plan characterized by full Pareto efficiency.
and no shortages. (Implications of shortages are considered later.) Starting from an equilibrium with no second economy, let us consider what happens when illegal firms emerge. As these firms can operate more efficiently than enterprises, they are able to offer higher wages to workers than enterprises. The firms are also able to pay a higher price for capital to enterprise managers than what the latter are charged by the state in the form of surplus or profits. Thus, incentives for diversion of resources from the first to the second economy exist. Of course, the state must monitor this diversion; otherwise all resources will move into the second economy. In the simple case where the state monitors only illegal firms but not enterprises, it turns out that at the initial shadow prices the parallel market does not alter the allocation of resources between the two goods. But due to higher efficiency, private production serves to increase the total supply of the labor intensive good. As the state continues to want to achieve the demand-supply balance, it has to lower the price of the labor intensive good and adjust the associated shadow prices of labor and capital. Changes in the latter are such that wages fall and profits rise. In addition, the state receives revenues in the form of penalties paid by the firms caught operating illegally. In the absence of any redistribution of income, workers are hurt while bureaucrats who derive their income from state revenues benefit. Of course, the overall income in the economy rises and it is possible to make both groups better off through appropriate transfers. Also, if illegal firms produce the capital intensive good, changes in shadow prices will benefit the workers.

The second major source of the parallel market considered in the paper is the presence of shortages in the initial equilibrium. It is assumed that the state implements a plan which is characterized by a larger production of the capital intensive good than is necessary to ensure Pareto efficiency and
demand-supply balance. At the official price, there is a shortage of the labor intensive good and the state has to resort to quantitative allocations. (See Grossman (1983) for evidence in this regard.) The shortage implies that illegal production is profitable so long as resources can be obtained at prices that are not much higher than those paid by enterprises. Here again it turns out that if enforcement is directed against illegal firms and not enterprises, parallel production does not alter the overall allocation of resources between the two sectors. Moreover, if production costs are the same in the official and second economies, total outputs of the two goods remain unchanged as well. All that happens is that production of the labor intensive good moves partially underground and the good is sold at a higher price in the second than in the first economy. Of course, if illegal firms are more efficient, they will serve to increase the total availability of the good which is potentially beneficial.

A broad conclusion of the paper is that in analyzing parallel markets, general-equilibrium considerations play an important role. Partial equilibrium models often run the risk of ignoring the overall resource constraints and the interaction between the first and second economies. The paper demonstrates that adjustments in the official production plan and the associated shadow prices in response to the emergence of parallel markets can lead to results that are dramatically different from those implied by partial equilibrium reasoning.
Figure I: A Schematic Flow Chart of First and Second Economies
THE PARALLEL MARKET IN CENTRALLY PLANNED ECONOMIES

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Abstract

This paper presents some general-equilibrium models of the parallel market in centrally planned economies. The models are based on the hypotheses that private firms can operate more efficiently than bureaucratically-run state enterprises and that Soviet-type economies are characterized by price controls and quantitative allocations. The state's enforcement policy is explicitly modeled. Although the welfare implications of the parallel market for workers are ambiguous in general, under a variety of circumstances they turn out to be negative. For instance, responding to the shortage created by price controls, illegal firms divert resources from the official economy into the parallel market. The result is a higher price in the parallel market without any increase in the total supply of the good.

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The analysis of the "parallel" or "second" economy has become a matter of urgency in view of the increasing recognition of its importance in many economies. The phenomenon was widely noted and discussed in the context of the extensive control-generated transactions in developing countries, as in India (e.g., Bhagwati and Desai, 1970). It has recently been the subject of a growing empirical literature in the context of centrally planned economies (CPEs), with notable contributions by Simes (1975), Grossman (1977) and Katsenelinboigen (1977) on the importance of the parallel economy in the Soviet Union.

Although an important theoretical literature exists, incorporating parallel markets into conventional general-equilibrium analysis of market economies in an essential fashion (Bhagwati and Hansen, 1973; Sheikh, 1974; Pitt, 1981; and Martin and Panagariya, 1984;), the analysis there does not reflect any of key aspects of CPEs.1 Addressing the issue in the context of CPEs, however, Wellisz and Findlay (1986) have recently undertaken a pioneering parallel-market analysis, drawing on the tools and insights from the earlier literature for market economies.2

In the following analysis, I offer an alternative formalization of parallel markets in CPEs, which departs radically in conception from the Wellisz and Findlay model and, I may suggest, captures in a more natural fashion the essence of the problem at hand. This formalization is based on two important features central to the emergence of parallel markets in CPEs. First, the analysis incorporates the widely-held idea among Western experts that profit-seeking private firms can operate technically more efficiently than the bureaucratically-run state enterprises.3 Second, the paper takes explicit account of the fact that Soviet-type economies are subject to price controls and quantitative allocations which provide incentive for underground economic activity.
An important issue that any general-equilibrium analysis of CPEs must confront concerns the disposal of the income generated by capital. In these economies, capital (the so-called "means of production") as well as the income attributable to it belongs to the state. Therefore, we must specify how the state spends this income. In reality, a large part of the state's civil outlays in the Soviet Union goes into either capital formation or the provision of public services. For this reason, the most realistic approach will be to assume that the income attributable to capital is spent either on capital accumulation or on public goods. Unfortunately, both of these assumptions lead to considerable complications that are not central to the analysis of parallel markets. The former assumption requires a dynamic specification of the model while the latter one introduces the usual difficulties associated with the treatment of public goods.

Beyond capital formation and provision of public goods, the leading category of non-military governmental expenditures in the Soviet Union is administrative services. Given this fact and in view of the difficulties with the alternative assumptions discussed in the previous paragraph, I shall assume that the income generated by capital is used to pay the salaries of bureaucrats. Thus, the population will be viewed as being divided into two groups, workers in industrial enterprises and bureaucrats in the state apparatus. The workers will be assumed to derive their income from wages only while bureaucrats receive the state's income.

Section 1 states and examines a CPE model, assuming that the parallel market does not exist. In spirit, this model resembles the Lange (1936)-Lerner (1934) idealization of the Soviet economy. At this stage, no price-quantity restrictions are introduced. Section 2 introduces the parallel market with illegal firms utilizing productive factors diverted from enterprises in the legal, planned economy but with enforcement expenditures
directed at the illegal firms. The model in this section is driven by the fact that private firms can operate more efficiently than state enterprises. Section 3 extends the analysis to the realistic case where the state seeks to monitor the activities of legal enterprises aimed at seeking central allocations of the factors of production for profitable diversion to illegal firms. Section 4 studies the parallel market in a setting where the initial equilibrium is characterized by quantitative allocations at controlled prices and, consequently, the economy suffers from imbalances in demand and supply. Finally, Section 5 provides a summary of the paper.

1. An Initial Equilibrium

Consider a centrally planned economy endowed with two factors of production, labor (L) and capital (K). The economy produces two goods, 1 and 2, via production functions that are linear homogeneous in the two inputs. Good 1 is relatively labor intensive.

Assume that labor is supplied by private individuals while capital is owned by the state and that the income generated by each factor accrues to its owner. As mentioned in the introduction, the state's income is used entirely to pay the salaries of bureaucrats. Let the utility functions of workers as well as bureaucrats be homothetic. Bureaucrats have a stronger preference for the capital intensive good than workers. That is to say, for a given marginal rate of substitution, bureaucrats' demand for good 2 relative to good 1 is higher than that of workers.

In order to analyze the implications of the parallel economy, we need to establish an initial equilibrium. I will postpone the discussion of the implications of price controls and quantitative allocations until Section 4. Presently, I will proceed by assuming that the state's planning agency chooses various shadow prices so as to establish an equilibrium characterized by full Pareto efficiency. Thus, the shadow price of good i, \( p_i \) (\( i = 1, 2 \)), the shadow
rental on capital, \( r \), and the wage rate, \( w \), satisfy the relationship

\[ p_i = c_i(w, r) \quad i = 1, 2 \quad (1) \]

where \( c_i(w, r) \) represents the average and marginal cost of producing good \( i \).

Given linear homogeneous production functions, the \( c_i(w, r) \) will also be linear homogeneous in their arguments. Therefore, we can solve the marginal-cost-pricing conditions to obtain \( w \) and \( r \) as linear homogeneous functions of \( p_1 \) and \( p_2 \). Formally, we can write \( w = w(p_1, p_2) \) and \( r = r(p_1, p_2) \). These two equations allow us to represent the economy's GDP by \( r(\cdot)K + w(\cdot)L \). The first partial of this GDP function with respect the \( i \)th price yields the equilibrium output of the \( i \)th good. Letting \( X_i \) denote the output of good \( i \), we have

\[ X_i = r_i(p_1, p_2)K + w_i(p_1, p_2)L \quad i = 1, 2. \quad (2) \]

where \( r_i(\cdot) \) and \( w_i(\cdot) \), respectively, denote the first partials of \( r(\cdot) \) and \( w(\cdot) \) with respect to the \( i \)th argument.

Equations (1) and (2) summarize the production side of our economy.

Next, let us consider the demand side. As noted earlier, bureaucrats as well as workers have homothetic preferences. Denoting the (government) bureaucrats' and workers' demands for good \( i \) by \( G_i \) and \( D_i \), respectively, and making use of the homotheticity assumption, we can write

\[ G_i = r(p_1, p_2)K/g_i(p_1, p_2) \quad i = 1, 2 \quad (3) \]

\[ D_i = w(p_1, p_2)L/d_i(p_1, p_2) \quad i = 1, 2 \quad (4) \]

where \( g_i(\cdot) \) and \( d_i(\cdot) \) are linear homogeneous and their first partials, \( g_{ij}(p_1, p_2) \) and \( d_{ij}(p_1, p_2) \), are positive for \( i = j \) and negative for \( i \neq j \).

In intuitive terms, \( p_i/g_i(\cdot) \) represents the proportion of bureaucrats' income spent on good \( i \). A similar interpretation applies to \( d_i(\cdot) \). Remembering that bureaucrats have been assumed to have a stronger preference for good 2 than workers, we must have \( G_2/G_1 > D_2/D_1 \) or, equivalently, \( g_1/g_2 > d_1/d_2 \).

We can now close the model by invoking the usual market-clearing
condition

\[ X_1 = G_1 + D_1 \]  \hspace{1cm} (5)

The system represented by (2)-(5) consists of 7 equations in 7 endogenous variables, namely, \( X_1, X_2, G_1, G_2, D_1, D_2 \) and \( p_1/p_2 (=p) \). We can substitute for \( X_1, G_1, \) and \( D_1 \) from (2)-(4) into (5) and solve the resulting equation for \( p_1/p_2 \). Once we have \( p_1/p_2 \), we can substitute it back into (2)-(4) to obtain the equilibrium values of the remaining variables. Throughout the paper we will think of good 2 as the numeraire good. In our graphical analysis, we will explicitly set \( p_2 = 1 \) and \( p_1 = p \), although in the algebraic analysis \( p_1 \) and \( p_2 \) will appear separately for ease of exposition.

The equilibrium just described will be called the "initial" equilibrium. In Figure 1, \( TT' \) shows the economy's transformation curve. \( U_1^G \) and \( U_2^G \) represent bureaucrats' indifference curves while \( U_1^D \) and \( U_2^D \) show workers' preferences. Note that if rental income is redistributed to workers, the equilibrium will obtain at point \( C \). Similarly, if all income is spent according to bureaucrats' preferences, equilibrium will be given by point \( B \). In the present case, the equilibrium production point will be somewhere between \( C \) and \( B \). Let this point be represented by \( \tilde{L} \). The equilibrium price ratio, \( \tilde{p} \), will equal the slope of the transformation curve at \( \tilde{L} \). Given this price, we can uniquely determine the ratios in which the two goods are demanded by bureaucrats and workers. These ratios are shown by rays \( OG \) and \( OD \). Constructing the parallelogram \( OGLD \), we can obtain \( \tilde{G} \) and \( \tilde{D} \) as the consumption points of bureaucrats and workers, respectively. It is evident that the precise location of points \( \tilde{L}, \tilde{G} \) and \( \tilde{D} \) will depend on the distribution of GDP between bureaucrats and workers. The larger the bureaucrats' share in income the closer will \( \tilde{L} \) and \( \tilde{G} \) be to \( B \) and farther will \( \tilde{D} \) be from \( C \).

It is useful to conclude this section by summarizing the institutional arrangements in the initial equilibrium. We have postulated an economy in
which capital is owned by the state. The rental on capital accrues to the state while wage income goes to workers. The state’s income is used entirely for paying the salaries of bureaucrats. Bureaucrats have a stronger preference for capital intensive goods than workers. Based on its knowledge of tastes, technology and endowments, the central planning agency determines the appropriate shadow prices for goods and inputs and tells the state enterprises to follow the marginal-cost-pricing rule. Labor is paid according to the shadow wage rate determined by the planning agency while bureaucrats receive the residual. The output is then sold through the state’s retail shops at the shadow prices just mentioned. 10

2. A Simple Parallel-Market Model

In this section, I will introduce production activities of illegal firms into the economy just outlined. As noted earlier, a description of these activities can be found in the studies by Simes (1975), Katsnelinboigen (1977), Grossman (1977) and others. Of particular interest in the present context is the following excerpt from Grossman.

Last, there are the underground entrepreneurs in the full sense of the term: that is, individuals who promote and organize production on a substantial scale, employ the labor of others, obtain materials and machinery on the black market, and distribute their output widely...The products involved are often consumer goods (garments, footwear, household articles, knickknacks, and the like) but can be producer goods as well.

In order to model illegal activities properly, it is important to introduce the government’s enforcement policy explicitly. I shall consider two types of enforcement policies. In this section, enforcement will be directed against illegal firms while in Section 3, it will be directed against enterprises who seek the state’s resources for diversion to the parallel
As noted in the introduction, I shall draw upon the popular notion among
the Western Sovietologists that private firms which are motivated by profit
seeking do not suffer from the kind of X-inefficiency that bureaucratically
run state enterprises do. I shall capture this notion by assuming that there
are potential private entrepreneurs who can produce good 1 more efficiently —
in Hicks-neutral sense — than state enterprises. In formal terms, these
entrepreneurs will be assumed to face the unit-cost function
\[ c^P_1(w, r) = (1 - \alpha) c_1(w, r) \] (6)
where "P" stands for "parallel" and \( \alpha \) is a constant between 0 and 1. Recall
that \( c_1(\cdot) \), introduced in equation (1), is the unit-cost function facing state
enterprises in sector 1.

It is evident that given fixed endowments of labor and capital, illegal
firms will have to rely on state enterprises for their input supplies. From
the viewpoint of modeling, this fact raises the important question: What is
the mechanism by which resources are diverted from state enterprises to the
parallel market? In what follows, I attempt to answer this question in the
simplest possible manner.

Consider first the planning agency's problem. Taking the enforcement
policy as given, the planning agency chooses the shadow prices of goods and
inputs so as to ensure full employment of resources and equality of demand for
and supply of goods. It then allows workers to allocate themselves freely at
the chosen shadow wage and provides enterprises with as much capital as they
demand at the shadow rental rate.

Given that illegal firms are technically more efficient than state
enterprises, they can offer a higher wage than the latter. Therefore, labor
is attracted to the parallel economy. Similarly, illegal firms can offer a
higher rental on capital than that charged by the state. This fact encourages
enterprise managers to obtain more capital from the state than they need and divert it to the parallel market.

It is evident that if the state did not have an enforcement policy, illegal firms will wipe out state production entirely. Therefore, it is essential to introduce the state's enforcement policy explicitly. I shall assume that the state maintains an enforcement agency entrusted with the task of apprehending illegal firms. For simplicity, it will be assumed that the enforcement agency has a fixed quantity of resources at its disposal. These resources are separate from the labor and capital used in goods production.

Let us denote the probability of detection of a firm by q. A key assumption of our analysis will be that q is a positive function of the economy-wide ratio of illegal-to-total output of good 1, \( x \). Thus, we will write

\[
q = q(x) \quad 0 \leq x \leq 1; \quad q(0) = 0 \text{ and } q(1) = 1.
\]

\[
q'(x) > 0 \text{ for } 0 < x < 1.
\]

The assumption \( q'(x) > 0 \) is necessary to ensure coexistence of legal and illegal production and can be justified on the ground that the vigor of the state's enforcement policy is a positive function of the ratio of illegal output to the total output. Alternatively, we could argue that as the proportion of illegal output expands, illegal firms become more visible to the enforcement agency. For simplicity, we will assume that the penalty on a firm caught operating illegally is complete confiscation of its output.

We can now proceed to determine \( x \), the ratio of illegal-to-total output. Let the wage and rental rate paid by an illegal firm not apprehended by the enforcement agency be \( w^P \) and \( r^P \), respectively. Given our penalty assumption, the wage and rental paid by a firm which gets caught will, of course, be 0. Therefore, the expected wage and rental rate in the parallel market may be written \((1-q)w^P\) and \((1-q)r^P\), respectively. Assuming that workers and
enterprise managers are risk neutral, we obtain \((1-q)w^P = w\) and \((1-q)r^P = r\). Perfect competition among firms not caught will lead to \(c_1^P(w^P, r^P) = p_1\) or, equivalently, \(c_1^P(w, r) = (1-q)p_1\). This last equation, combined with (1) and (6), yields
\[
q(x) = a
\]  
(7)

Intuitively, the cost advantage enjoyed by private firms is exactly offset by higher input prices facing them.

It is worth emphasizing that the equilibrium value of \(x\) turns out to be a function of \(\alpha\) alone. Interestingly, it does not depend even on the relative price ratio. The reason for this result is that neither the proportionate cost advantage to illegal firms nor the proportionate penalty in terms of higher input prices depends on the price ratio.

In order to determine the equilibrium values of other variables, let us introduce the GDP function. The GDP in the present economy consists of factoral incomes plus the revenue raised by the state via penalties. As the expected factor prices in illegal sector equal actual factor prices facing state enterprises, we can represent the GDP by \(r(.)K + w(.)L + q(.)p_1.x.X_1\). Note here that \(x.X_1\) is the output produced in the parallel economy.

Differentiating the GDP function partially with respect to prices, we obtain total outputs.
\[
X_1 = r_1(p_1, p_2)K + w_1(p_1, p_2)L + q(x)x.X_1
\]  
(8)
\[
X_2 = r_2(p_1, p_2)K + w_2(p_1, p_2)L
\]  
(9)

On the demand side, workers' demand function continues to be given by equation (4). Assuming that the state treats the revenue generated by the enforcement agency in the same way as rental income, bureaucrats' demand function, (3), must be replaced now by
\[
G_i = \frac{r(p_1, p_2)K + p_1q(x)x.X_1}{g_i(p_1, p_2)} = 1,2
\]  
(3')

Equation (5) continues to give the market-clearing condition.
The system embedded in (7)-(9), (3'), (4) and (5) consists of 8 equations and can be solved for 8 endogenous variables, \( x, X_1, X_2, G_1, G_2, D_1, D_2 \) and \( p_1/p_2 \). We shall refer to the solution to this system as parallel equilibrium.

At this point, it is useful to comment briefly on the output equations (8) and (9). Comparing them with equations (2), it is evident that at a given price ratio, the allocation of labor and capital between goods 1 and 2 in the initial and parallel equilibria is exactly identical. The reason for this result is that for a given price ratio, the (expected) wage and rental rate which guide the allocation of resources between the two goods is the same in the two equilibria. As long as state enterprises continue to produce good 1, the wage and rental rate facing state enterprises and hence the expected wage and rental rate in the parallel economy continue to be determined by equations (1). Of course, the higher technical efficiency of private firms implies a net gain in the output of good 1. More precisely, given the cost differential shown in equation (6), production of quantity \( x.X_1 \) in the parallel market results in a sacrifice of only \((1 - \alpha)x.X_1\) amount in the "First Economy". Therefore, the presence of private firms leads to a net addition of \( \alpha.x.X_1 \) (=q(.\()x.X_1\) amount to the total quantity of good 1.

Figure 2 compares the initial and parallel equilibria graphically. For notational convenience, I use tilde (\(\tilde{\cdot}\)) and hat (\(\hat{\cdot}\)) to distinguish the equilibrium values of various variables in the initial and parallel equilibrium, respectively. Point \(\tilde{L}\) represents the production point while \(\tilde{G}\) and \(\tilde{D}\) are consumption points in the initial equilibrium. Holding the price at \(\tilde{p}\), let us introduce the parallel market. Since \(q(0) = 0\) and \(\tilde{p}_1 = c_1(\tilde{w}, \tilde{r})\), the expected profit on the first unit of illegal production, \([1-q(0)]\tilde{p}_1 - (1 - \alpha).c_1(\tilde{w}, \tilde{r})\), is positive. Resources will be drawn into the parallel market and due to the higher technical efficiency of private firms, production will move out of \(\tilde{TT}'\). Holding the price ratio temporarily fixed at \(\tilde{p}\), the parallel
market activity will push the economy's production point directly to the right of \( L \) to, say, \( R \). At point \( R \), workers' income is unchanged while that of bureaucrats increases by the amount of penalties, \( LR \). As the bureaucrats will spend only a part of their income from penalties on good 1, there will be an excess supply of that good at price \( \tilde{p} \). It follows that the price of good 1 will fall and the economy will move from \( R \) in the northwestern direction. In the parallel equilibrium, production will take place at a point such as \( \hat{P} \). The new equilibrium price, \( \hat{p} \), will be given by the slope of \( \tilde{TT}' \) at point \( \hat{Q} \) where \( \hat{Q} \) lies horizontally to the left of \( \hat{P} \). The state will collect \( \hat{P}Q \) in penalties. As \( \hat{p} \) is less than \( \tilde{p} \), the consumption rays will flatten down to \( OG' \) for bureaucrats and \( OD' \) for workers. The new consumption points, \( \hat{G} \) and \( \hat{D} \), are obtained by completing the parallelogram \( \hat{O}\hat{G}\hat{P}\hat{D} \).

The most striking result is that the presence of the parallel market is detrimental to the welfare of workers and beneficial to bureaucrats. The reason for this result is that the presence of the parallel market causes the price of good 1 to fall so that the real income of labor falls unambiguously via the Stolper-Samuelson effect. By contrast, bureaucrats' income rises due to an increase in the rental price of capital as well as the revenues raised in terms of penalties.

This result leaves the question of the overall effect of the parallel market on social welfare unanswered. There are essentially three possibilities in this regard. First, we could assume that social welfare depends solely on the welfare of workers. In this case, the parallel market will be detrimental to social welfare. Second, we could assume that social welfare depends on bureaucrats' utility. Under this assumption, social welfare will improve with the emergence of the parallel market. Finally and more realistically, social welfare may be assumed to depend on the welfare of both sets of consumers. In this case, the effect of the parallel market on
social welfare will be ambiguous in general. However, it can be shown that appropriate tax-subsidy schemes can be designed to improve the welfare of each entity in the presence of the parallel market. On this basis, one will conclude that the parallel market is (at least potentially) welfare improving.

Our model can be explored further by considering the comparative statics effects of a change in the enforcement policy. Formally, this task is accomplished by shifting the \( q(.) \) function. Without presenting all the details of this exercise, it is worthwhile to note that a stricter enforcement in the present model leads to an increase in the official GDP. This result is consistent with the recent Soviet experience. For instance, in the late 1970's Brezhnev's policy to control the parallel market was ineffective. When Andropov came to power, he cracked down on this market and the officially reported output showed a significant jump. This experience was repeated when Gorbachev replaced the weak leadership of Chernenko.

Before concluding this section, it is worthwhile to outline two generalizations of our model. First, the model is easily modified to allow for parallel markets in both sectors. In this case, introduction of parallel markets at a given price ratio will lead to an expansion of both goods. In terms of Figure 2, point \( R \) will lie strictly northeast of \( L \). Depending on whether this point happens to be above or below ray \( OL \) (not drawn), the shadow price of good 1 will rise or fall. In the former case, the real wage will rise unambiguously. Thus, once we allow for parallel markets in both sectors, it is possible for the welfare of workers to rise. Furthermore, since the state gets to collect penalties, bureaucrats' welfare could also improve despite a decline in the real value of rental income.

The second extension concerns inclusion of enforcement costs into the model. This task can be accomplished most simply by assuming that the probability of detection also depends on the number of inspectors (i.e.,
amount of labor) engaged in enforcement activity. In this setting, a Rybczynski effect will accompany the introduction of the parallel market. Thus in Figure 2, at the initial prices, the economy will end up at a point northeast of R. If enforcement employs sufficiently large amount of labor, the net result may be a shortage of good 1 at the initial prices and the planning agency will have to raise the relative price of that good in order to clear the market. Once again, we will have a reversal of the results derived from the simple model.

3. Penalties on "Resource Seeking"

In the last section, it was assumed that the state does not monitor the resource-seeking activities of enterprises. We will now modify this assumption and introduce the monitoring of resource-seeking activities of enterprises. In order to keep matters simple, I will drop the enforcement activity directed at illegal firms and concentrate exclusively on the monitoring of enterprises. It may be pointed out at the outset that my objective here is to merely indicate the manner in which the monitoring of enterprises can be incorporated into the analysis. Therefore, I will not pursue the implications of such monitoring in detail.

For simplicity, let us assume that resource-seeking activities take place in sector 1 only. It is a fairly straightforward matter to extend the analysis to allow for resource-seeking in both sectors. The state can monitor enterprises in at least two ways. First, based on the knowledge of production technology and enforcement policy, it can form an idea of the capital-output ratio in actual production. Comparing this ratio with the officially reported one, it can arrive at an estimate of the amount of capital diverted to the parallel market. Second, the state can conduct on-site inspections to detect discrepancies between actual volume of capital and that on official records.

The outcome of the state's enforcement policy depends critically on
whether it monitors the diversion of capital at the aggregate level or at the level of individual enterprises. In the former case, an individual enterprise has no control over the probability of detection whereas in the latter case, its actions do influence that probability. I will consider briefly each type of monitoring.

In the case when the state is able to monitor only the aggregate capital-output ratio, raids are conducted randomly. Therefore, we can assume that all enterprises face a uniform probability of detection. In order to ensure an interior solution, we must further assume that this probability is an increasing function of the overall ratio of diverted-to-total capital in sector 1. The assumption can be justified on the ground that the frequency of raids is an increasing function of diverted-to-total capital in sector 1. Upon detection, a penalty is assessed on the diverted capital at the ad valorem rate $\gamma$.

I will assume that as a part of their resource-seeking strategy, enterprise managers replace the planning agency's directive to equate the marginal cost to price by the profit-maximization objective. Using superscripts $F$ and $P$ to distinguish the variables in the first and parallel economies, respectively, we can represent the $j$th enterprise's expected-profit function by

$$
\rho_j = p_1 G_1(K_{1j}^F, L_{1j}^F) + (r_P - r)K_{1j}^P - rK_{1j}^F - wL_{1j}^F - \gamma \cdot r \cdot m(K_{1j}^P/K_1)K_{1j}^P
$$

where $G_1(.)$ is the production function facing enterprises in sector 1, $r_P$ is the rental rate in the parallel market, $m(.)$ is the probability of getting caught diverting capital to the parallel market, and $K_1 = K_1^F + K_1^P$. The first term in the profit function represents the revenue earned on the output produced. The second term is the profit from diverting capital to the parallel market. The third and fourth terms are costs of producing the official output. Finally, the last term represents the expected penalty.
Maximization of $p_j$ with respect to $L_{1j}^F$, $K_{1j}^F$ and $K_{1j}^P$, respectively, yields

$$p_1 \partial G_1 (K_{1j}^F, L_{1j}^F) / \partial L_{1j}^F = w$$  \hspace{1cm} (10a)$$

$$p_1 \partial G_1 (K_{1j}^F, L_{1j}^F) / \partial K_{1j}^F = r$$  \hspace{1cm} (10b)$$

$$r^P - r = \gamma . r . m (K_{1j}^P / K_{1j}^T)$$  \hspace{1cm} (10c)$$

The first two of these conditions are standard. They imply that unit-cost pricing, shown in equation (1), continues to hold. According to condition (10c), competition among enterprises to divert capital leads to an equalization of the difference between the official and private rental rates and the penalty per-unit of diverted capital. The extra rental earned by capital in the parallel market flows back to the state in the form of penalties.

Conditions (10a)-(10c) hold for all firms. Therefore, we can drop subscript $j$ and apply the resulting conditions at the industry level. These three conditions along with two similar conditions each for the parallel market and sector 2, two full employment conditions, and three production functions sum up to 12 equations which determine the supply side of the economy. Given factor endowments and commodity prices, they can be solved for the 12 endogenous variables which include three outputs ($X_1^F$, $X_1^P$ and $X_2$), three labor allocations ($L_1^F$, $L_1^P$ and $L_2$), three capital allocations ($K_1^F$, $K_1^P$ and $K_2$), legal wage and rental rates ($w$ and $r$) and the parallel market rental rate ($r^P$). The model can be closed by adding the demand side in the usual fashion.

Some of the important implications of this model can be derived in a straightforward manner. First, since the rental on capital facing private firms is higher than that facing state enterprises, the rates of technical substitution in the two sectors will fail to equalize. Second, at a given price ratio, an increase in the average rental rate in sector 1 will lead to a higher labor-capital ratio in that sector than the one obtained in the initial equilibrium of Section 1. Finally, as long as the presence of the parallel
market leads to a decline in the relative price of good 1, the income
distribution results derived in the last section will continue to hold. The
reason for this last conclusion is that conditions \( p_1 = c_1(w, r) \) continue to
hold in the present model so that the wage and rental rates remain tied to the
goods prices in the usual Stolper-Samuelson manner.

Finally, let us consider briefly the case when the state monitors the
capital-output ratios of individual enterprises so that the probability of
detection depends on the enterprise-level ratio of diverted-to-total capital.
Formally, in the profit function given by \( \rho_j = m(k^P_{1j}/k_{1j}) \) must be replaced by
\( m(k^P_{1j}/k_{1j}) \). Making this substitution and maximizing \( \rho_j \) with respect to \( L_{1j}^F, K_{1j}^F \) and \( K_{1j}^P \), we have the following conditions.

\[
\begin{align*}
  p_1 \frac{\partial G_1(K^F_{1j}, L_{1j}^F)}{\partial L_{1j}^F} &= \omega \\
  p_1 \frac{\partial G_1(K^F_{1j}, L_{1j}^F)}{\partial K_{1j}^F} + \gamma r m'(k^P_{1j}/k_{1j}) (k^P_{1j}/k_{1j})^2 &= r \\
  r^P - r &= \gamma r m(.) + \gamma r m'(k^P_{1j}/k_{1j}) (k^P_{1j}/k_{1j})
\end{align*}
\]

Condition (11a) is standard and needs no further comment. Condition (11b) has
one extra term. This term captures the favorable effect of increasing \( K_{1j}^F \) on
the probability of detection. According to (11c), the extra revenue earned by
diverting one more unit of capital to the parallel market must equal the
penalty on this unit (=γr m) plus the increase in penalties
(=γr m'.(k^P_{1j}/k_{1j})) on the pre-existing units due to an increase in the
probability of detection.

In the present model, enterprises are treated symmetrically. Therefore,
(11a)-(11c) apply to all enterprises. If we drop subscript \( j \), the resulting
equations will give us industry-level equilibrium conditions in sector 1 of
the First Economy. Once again, these three conditions can be combined with
the other equations of the model in the manner described for the previous case
to obtain the parallel market equilibrium.
4. Price Controls and Quantitative Allocations

The initial equilibrium introduced in Section 1 provides the simplest possible framework necessary to analyze the parallel market activities arising due to higher efficiency of private firms. A possible criticism of this equilibrium is that it ignores the existence of price controls and quantitative allocations that are so characteristic of Soviet-type economies. In the present section I will modify the analysis to allow for these important features of CPEs. In particular, I will present a parallel market model based on Grossman’s (1983) observation on the Soviet economy that in the consumer goods sector "the official fixed individual retail prices tend to deviate from their actual equilibrium levels and, more often than not, tend to be too low."

Let us begin the analysis by outlining what we will call a "controlled" equilibrium. Assume for the moment that there are no illegal firms and that the state wishes to implement a plan which involves a smaller proportion of the labor intensive good than at $\hat{L}$ in Figure 1. To keep matters simple, let this plan be represented by the point where bureaucrats' indifference curve is tangent to the production possibilities frontier. The results remain unchanged if the state chooses any other plan so long as the plan involves a smaller proportion of good 1 than at $\hat{L}$.

Using a bar ($\bar{}$) to distinguish variable values at the controlled equilibrium, the production plan may be represented by point $\bar{B}$ in Figure 3. Denote the shadow prices associated with this plan by $\bar{p}_1/\bar{p}_2 (= \bar{p})$, $\bar{w}$ and $\bar{r}$. At $\bar{p}$, bundle $\bar{B}$ will translate into $\bar{OY}$ of national income in terms of good 2. Letting $\bar{OY}_G$ be bureaucrats' income, draw line $\bar{Y}_G V$ with a slope $\bar{p}$. Intersection of this line with ray $OG$ at $\bar{G}$ will give the bureaucrats' consumption bundle. Subtracting the bureaucrats' income from total income and their consumption bundle from total output bundle, we can obtain $\bar{OY}_D$ and $\bar{D}$, respectively, as the income and consumption bundle of workers. Since $\bar{OD} = \bar{BG}$ (or equivalently $\bar{OG} = \bar{DB}$) and $\bar{OY}_D = \bar{Y}_G \bar{V}$ (or $\bar{OY}_G = \bar{Y}_D \bar{V}$) by construction, the
line joining $\bar{Y}_D$ and $\bar{D}$ will have a slope equal to $\bar{p}$. Thus, if the state sells good 1 to workers at $\bar{p}$, bundle $\bar{D}$ will exactly exhaust private income.

Remembering that workers' tastes are biased in favor of good 1, however, the marginal rate of substitution at $\bar{D}$ will exceed $\bar{p}$. Put differently, there will be a shortage of good 1 at the official price $\bar{p}$. This shortage will pave the way for the parallel market to emerge.

In considering the parallel market in this section, I shall limit the analysis to the relatively simple enforcement policy employed in Section 2. Thus, the state will be hypothesized to prosecute only the firms operating illegally. The resource diversion activity of enterprises will not be subject to monitoring.

Initially assume that private firms do not differ from state enterprises in terms of production efficiency. At the initial equilibrium, the probability of detection is zero ($q(0) = 0$). Therefore, the wage and rental rate in the illegal sector at that point will be the same as that in the legal sector. The unit cost of production of the first illegal unit of good 1 will be $\bar{p}$. Remembering that at the initial equilibrium workers are willing to pay a price higher than $\bar{p}$, the potential expected profits from selling the first unit illegally are positive. The parallel market will emerge.

As illegal production expands in relation to legal production, the probability of detection will rise and the wage and rental rate paid by the successful firms in the parallel economy will have to rise to keep the expected input prices there equal to $\bar{w}$ and $\bar{r}$. Moreover, the relative price of good 1 in the parallel market, $p^P$, will decline as illegal production expands. In equilibrium, $p^P$ will equal the average cost of production in the parallel market. As in Section 2, we have $(1-q)w^P = \bar{w}$ and $(1-q)r^P = \bar{r}$ so that $p^P = c_1(w^P, r^P)$ which leads to

$$p^P[1 - q(x)] = c_1(\bar{w}, \bar{r}) = \bar{p}$$

(12)

Introduction of the parallel market in the economy depicted in Figure 3
gives rise to two additional variables, \( x \) and \( p^P \). In order to determine the equilibrium values of these variables, we need two extra equations. The first of these equations is provided by (12) while the second one is obtained by setting the excess demand in the parallel market equal to zero.

In order to economize on space, I will not present the complete model algebraically. Instead, I will proceed directly to intuitive and graphical analysis. The first step in this direction is to note that if the state wants the economy as a whole (including the parallel market) to produce at \( \bar{B} \) in Figure 3, the planning agency must continue to set its shadow prices at \( \bar{p} \), \( \bar{w} \) and \( \bar{r} \). This observation follows from the fact that at \( \bar{p} \), \( \bar{w} \) and \( \bar{r} \), the entire economy's GDP function in terms of planners' prices remains unchanged at \( \bar{w}(\bar{p}_1, \bar{p}_2)L + \bar{r}(\bar{p}_1, \bar{p}_2)K \). Intuitively, as in Section 2, for a given price ratio the introduction of the parallel market does not alter the overall allocation of resources between sectors 1 and 2.

This result allows us to deduce immediately that if the state continues to guide the economy's total production towards \( \bar{B} \), introduction of the parallel market will once again lead to a decline in the utility of workers. As in Sections 2 and 3, the parallel market redistributes income from workers to bureaucrats. The latter continue to receive \( \bar{r}K \) in rental income and, in addition, collect \( q(.)xX_1 \) in penalties. From workers' viewpoint the penalties work as a tax in that they raise the effective price of good 1 facing them. For this reason and because the parallel market does not serve to increase the total availability of good 1, workers' utility declines in the new equilibrium.

The parallel market equilibrium is depicted in Figure 4 where I have also reproduced the various points of interest from Figure 3 without necessarily reproducing the associated budget lines or indifference curves. As already explained, the economy's total-output bundle continues to be \( \bar{B} \). Using a * to distinguish points associated with the parallel-market equilibrium, let us
represent the output of good 1 in the parallel economy by $\overline{BF}$ where $F$ lies horizontally to the left of $\overline{B}$. Note that $\overline{F}$ represents the output vector in the first economy. Suppose that penalties collected by the state from illegal firms amount to $\overline{FM}$. Then $\overline{BM}$ will be the quantity of good 1 sold illegally and $M$ will represent the basket disbursed through the official distribution system.

Penalties $\overline{FM}$ translate into $\gamma G \gamma$ amount of good 2. Note that this conversion is done at the parallel market price to be determined shortly. Recalling that the rental income has remained unchanged at $OY_G$, bureaucrats' income in terms of good 2 will rise to $OY_G$. As we will see shortly, workers' income valued at planners' prices will decline correspondingly.

Bureaucrats' consumption bundle can be obtained by drawing a line (not shown) through $Y_G$ with slope $\bar{p}$ and locating that line's point of intersection with ray $OG$. Thus, $G$ represents bureaucrats' consumption bundle. As expected, this bundle dominates bureaucrats' consumption bundle in the absence of the parallel market, $\overline{G}$.

Workers' consumption bundle is derived in two steps. First, subtract vector $OG$ from vector $OM$. This task is accomplished by completing the parallelogram whose two sides are given by $OG$ and $G M$ and obtaining point $C$. This point gives us the amount of good 1 obtained officially by workers at price $\bar{p}$. The second step is to add illegally purchased quantity $\overline{BM}$ of good 1 to that officially purchased and obtain $D$ as the consumption basket of workers. It is a straightforward matter to verify that $D$ will lie on ray $OG$ and that this basket will be dominated by $\overline{D}$, the basket consumed in the absence of the parallel market. We will also have $\overline{DD} = \overline{GG}$. At planners' prices, the value of basket $D$ will be below that of basket $\overline{D}$ exactly by the amount of penalty, $\gamma D \gamma := \gamma G \gamma$ where $\gamma D$ is obtained by drawing a line (not shown) with slope $\bar{p}$ through $D$.

Our remaining task is to determine the price of good 1 in the parallel
market. For this purpose, note that the factorial income received by workers continues to be given by \( OY_D \) in terms of good 2. Starting at \( Y_D \), workers can purchase bundle \( B^* \) at the official price, where \( B^* \) lies vertically above \( C \) and on line \( RY_D \). Note that \( RY_D \) has a slope equal to \( p \) and, as in Figure 3, it passes through point \( D \). Beyond \( B^* \), additional quantities of good 1 must be purchased at the higher illegal price. Since we have already determined \( D \) as the final consumption basket, we can conclude that the relative price of good 1 in the parallel market will be given by the slope of line \( B^*D^* \). Finally, the indifference curve of workers passing through \( D^* \) will be tangent to line \( B^*D^* \).

This analysis can be extended easily to incorporate the fact that private firms are more efficient than state enterprises. In this case, the production frontier in the presence of the parallel market shifts out horizontally. Consequently, the shadow price which maximizes bureaucrats' utility function will be below \( p \). The corresponding shadow wage and rental rates will be such that the real value of wage income will decline and that of rental income rise at planners' prices. Penalties on illegal firms which provide additional income for bureaucrats and raise the price of good 1 for workers will further benefit the former and hurt the latter.

Our model in this section is also modified easily to allow for the possibility that the state may choose the production plan so as to maximize bureaucrats' (or some other) utility function subject to the production possibilities in the official economy. Starting with production plan \( 
\bar{B} \) in Figure 3, introduction of the parallel market shifts the official production point horizontally to the left. At this production point bureaucrats's marginal rate of substitution exceeds \( p \) implying that the appropriate shadow price of good 1 is higher than \( p \). A higher shadow price of good 1 also implies a higher shadow wage and lower shadow rental rate. Thus, in this case it is possible for the parallel market to result in a higher utility for
workers provided penalties on illegal firms are not so large as to reverse the income distribution effect of a higher relative price of good 1.

4. Conclusions

In this paper, I have presented some general-equilibrium models of the parallel market in the context of centrally planned economies. The models are based upon notions that private firms can operate more efficiently than bureaucratically-run state enterprises and that Soviet-type economies are often characterized by price controls and quantitative allocations. The analysis is highly stylized and is intended to capture only the most important features of the parallel market as described by Katsenelinboigen, Grossman, and others. A key feature of the analysis has been to link the resource-seeking activities of state enterprises to the operations of private firms in the parallel market. The models also take an explicit account of the state's enforcement policy.

In the simplest model considered, the presence of the parallel market leads to a decline in the welfare of workers. The reason is that the additional goods supplied by more efficient private firms lower the relative price of labor intensive goods which, in turn, reduces the real wage. As workers derive their entire income by selling labor, their consumption possibilities necessarily decline. The overall impact of the parallel market is, of course, favorable in the sense that it is possible to design tax-subsidy schemes that will allow both bureaucrats and workers to benefit in the presence of the parallel market than in its absence.

The paper presents several modifications of the simple model. It is shown that if parallel markets exist in both sectors, it is possible for the income distribution result mentioned earlier to be reversed. Analogously, the presence of enforcement costs can give rise to the possibility that the parallel market raises workers' welfare.

The paper also develops a model in which the state chooses a plan which
creates a shortage of the labor intensive good. This setup naturally paves the way for profitable illegal production. The paper demonstrates that once again introduction of the parallel market proves harmful for workers. In the equilibrium that emerges, workers' wage income in terms of good 2 remains unaffected while they have to pay a higher price for illegally purchased quantities of good 1. The initial shortage of good 1 is cleared by a higher price (in the parallel market) rather than larger total supply.

According to our analysis, a more strict enforcement policy on the part of the state leads to an increase in the officially reported GDP. This conclusion is broadly consistent with reality as exemplified by significant jumps in the officially reported output after the weak governments of Brezhnev and Chernenko, respectively, were replaced by strong leadership of Andropov and Gorbachev.

An important conclusion which may be derived from this paper is that in analyzing the welfare implications of parallel markets for workers, general-equilibrium considerations can be quite important. Based on partial equilibrium reasoning, there is a general tendency to conclude that since the parallel market provides goods that are in short supply, its existence is necessarily beneficial for workers. This approach essentially disregards the overall resource constraints of the economy by assuming that the introduction of the parallel market will leave the official production entirely unchanged. The present paper demonstrates that adjustments in the official production plan and the associated shadow prices in response to the emergence of parallel markets can lead to results that are dramatically different from those implied by partial equilibrium reasoning.
Footnotes

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1 Nguyen and Whalley (1987) incorporate the parallel market into a computable general equilibrium model of the Indian economy.

2 Ericson (1983, 1984) analyzes the role of parallel markets in improving the allocation of intermediate inputs among state enterprises. Starting from a suboptimal allocation of intermediate inputs across enterprises by the planning agency, he demonstrates that the presence of parallel markets yields a Pareto superior allocation of intermediate inputs.

3 For example, Desai (1986) writes, "(T)he Soviet enterprises...are plagued by the Leibenstein disease of X-inefficiency..."

4 According to the evidence provided by Treml et al (1977), profits, generated mostly by state enterprises, constituted the largest single item in the state's income accounts in the year 1966. The turnover tax was the second largest item.

5 Once again, see Treml et al (1977) for empirical evidence.

6 Wellisz and Findlay assume that the state's income is spent the same way as private income. This assumption can be derived as a special case of ours if we assume that bureaucrats have the same preferences as workers.
I will not consider, however, the lobbying activities of the legal firms to secure more allocations. These types of directly-unproductive profit-seeking (DUP) activities have been considered under the rubric of "soft-budget constraints" by Kornai (1982) as discussed in Desai (1986, 1987). For a general discussion of DUP activities, see Bhagwati (1982).

The demand functions shown in (3) and (4) have a somewhat unconventional form. They can be derived as follows. Homotheticity implies $G_2/G_1 = h(p_1, p_2)$ where $h(.)$ exhibits zero degree homogeneity and $\partial h/\partial p_1$ and $\partial h/\partial p_2$ are, respectively, positive and negative. The state's budget constraint is $p_1 G_1 + p_2 G_2 = r(.)K$. Solving these two equations and defining $g_1 = p_1 + p_2 h(.)$ and $g_2 = (p_1 + p_2 h(.))/h(.)$, we obtain equation (3). Equation (4) can be derived analogously.

Note that a similar condition for good 2, $X_2 = G_2 + D_2$, exists but can be dropped in view of Walras's Law.

It may be noted that analytically speaking, the model considered in this section may be viewed as a special case of the two-individual, capital-labor model of income distribution pioneered by Johnson (1959).

Enforcement could also be directed against enterprises engaging in the diversion of capital. This task is left, however, for the next section.

The reason why this assumption is necessary to ensure coexistence of state enterprises and illegal firms can be explained as follows. If $q(.)$ were assumed to depend on the firm-level output, individual firms' marginal costs will be rising in their own output. Perfect competition will force each firm to produce an infinitesimally small output and we will obtain constant average and marginal costs at the industry level. Depending on whether these costs are below or above the average costs of state enterprises, illegal firms will supply the entire or no output. The assumption made here gives rise to an
external diseconomy at the firm level and hence rising average and marginal
costs at the industry level. For analogous assumptions in the smuggling
literature, see Bhagwati and Hansen (1973) and Sheikh (1974).

13Alternative penalties such as a partial confiscation or economic equivalents
of jail terms can be introduced without difficulty. But these alternatives
add little to the substance of our analysis.

14The welfare results, especially for workers, should be interpreted in terms
of expected welfare. Ex post, the workers employed in firms not caught
receive a higher wage than those employed in state enterprises and may benefit
as a result of the emergence of the parallel market.

15Strictly speaking, we must also assume that inspection at the time of raids
is so thorough that enterprises with a small proportion of diversion are as
likely to be caught as those with a high proportion. I am not entirely
comfortable with this assumption but have chosen to pursue the present case
because of its simplicity. The more realistic case where an enterprise can
influence its probability of detection by changing the enterprise-level ratio
of diverted-to-total capital is considered below.

16Alternatively, we could work with a separate preference function for the
state. As noted earlier, this will complicate the analysis without altering
the broad results.
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Figure 3
Figure 4