# Brussels-Lux Study Tour 2022
# K-12 Teacher Unit Planning Template

| Name: Stephanie Morgan | Unit Plan: Weeks of<br>Subject/Grade Level: NC Math 3 (10th & 11th grade) |
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| **Unit Title:** | Understanding the European Parliament & Exploring Allocation of MEPs |
| **Unit Narrative:** | Students will use their knowledge of linear, piecewise, polynomial, logarithmic, exponential, rational, and logistic functions to consider different ways to model the allocation of ministers in the European Parliament per member state. This will allow students to utilize math knowledge while developing an understanding of the European Parliament and how the number of ministers per member state is decided, which will also allow students to explore the Treaty on European Union (Article 14, Section 2) and begin to understand why degressive proportionality was utilized, making connections – where possible – to rationalizations used with American institutions (US House of Representatives). Using Desmos technology, students will explore degressive proportionality by analyzing the number of seats held by each member state in the European Parliament against the populations of each member state and address different representations and key features to explain the pros and cons of each model and justify one to use to predict potential MEPs allotted to member states that have just received candidate status (Ukraine and Moldova). Students will extend their mathematical knowledge through the interpretations of regressions and regression data provided to improve their mathematical reasoning and justifications. Students will then analyze their 'best' model and how well is models – or doesn't – the composition of the European Parliament pre-Brexit. Lastly, students will see how their predicted minister allocations would change, if at all, when run against pre-Brexit numbers and will discuss, using proper mathematical reasoning and vocabulary, why any changes observed may have occurred. |
| **Standards:** | *From North Carolina Standard Course of Study, North Carolina Math 3*:<br>**Seeing Structure in Expressions: *Interpret the structure of expressions***<br>NC.M3.A-SSE.1: Interpret expressions that represent a quantity in terms of its context.<br>NC.M3.A-SSE.1a: Identify and interpret parts of *piecewise*, absolute value, *polynomial, exponential,* and *rational* expressions including terms, factors, coefficients, and exponents.<br>NC.M3.A-SSE.1b: Interpret expression composed of multiple parts by viewing one or more of their parts as a single entity to give meaning in terms of a context.<br><br>**Creating Equations: *Create equations that describe numbers or relationships***<br>NC.M3.A-CED.1: Create equations and inequalities in one variable that represent absolute value, *polynomial, exponential,* and *rational* relationships and use them to solve problems algebraically and graphically.<br><br>**Interpreting Functions: *Interpret functions that arise in applications in terms of the context***<br>NC.M3.F-IF.4: Interpret key features of graphs, tables, and verbal descriptions in context to describe functions that arise in applications relating two quantities to include periodicity and discontinuities.<br><br>**Interpreting Functions: *Analyze functions using different representations***<br>NC.M3.F-IF.7: Analyze *piecewise*, absolute value, *polynomials, exponential, rational,* and trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities.<br><br>**Building Functions: *Build a function that models a relationship between two quantities***<br>NC.M3.F-BF.1: Write a function that describes a relationship between two quantities.<br>NC.M3.F-BF.1a: Build polynomial and exponential functions with real solution(s) given a graph, a description of a relationship, or ordered pairs (include reading these from a table).<br><br>*From North Carolina Standard Course of Study, North Carolina Math 4 (for extension)*:<br>**Understand how to model functions with regression**<br>NC.M4.AF.5.1: Construct regression models of linear, quadratic, exponential, logarithmic, and sinusoidal functions of bivariate data using technology to model data and solve problems<br>NC.M4.AF.5.2: Compare residuals and residual plots of non-linear models to asses goodness of fit of the model. |

**Objectives**

→ Review graph analysis and key features (domain and range, maximum and minimum, discontinuities, x- and y-intercepts) of different functions (linear, polynomial, rational, exponential, logarithmic, piecewise)

→ Understand the construction of the logistic model and how it relates to the key features defined for other functions

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- → Apply graph analysis and key features to a real-world application (apportionment of MEPs in European Parliament), identifying the parameters of the European Parliament with correct mathematical vocabulary and explanation
- → Use population data and the number of MEPs post-Brexit model data, discuss pros and cons of each model, understand how to interpret data given from Desmos, use a selected model to make predictions, and determine the validity of those predictions based upon parameters and restrictions given by the Treaty on European Union.
- → Understand the composition of the European Parliament, degressive proportionality, and consider the impacts of adding new member states (such as Ukraine and/or Moldova)

## Big Ideas

- → Real-world scenarios can be modeled using vocabulary and concepts relating to key features of graphs and graph analysis
- → Composition of the European Parliament can be understood and interpreted through mathematical analyses and predictions

## Essential Questions

- → What is the European Union?
- → What is the European Parliament?
- → What is degressive proportionality?
- → How can we model the number of Minsters of Parliament (MEPs) in the European Parliament, analyzing the model based on the parameters laid out in Article 14 Section 2, and predict the number of MEPs candidate states would gain?
- → How do we objectively prove that a mathematical model is the best representative of a set of data?

## Learning Acquisition and Assessment

| Students will know… (content/concepts) | Students will be able to… (skills, performance tasks) |
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| → The key features of graphs and how to interpret them within real-world applications<br>→ How to construct data models using Desmos and use those models to predict future events<br>→ Background on the European Union<br>→ How the composition of the European Parliament is decided and why | → Model data with Desmos using different function models and interpret the validity of that model with the coefficient of determination ($R^2$ or $r^2$) and residuals<br>→ Explain the composition of the European Parliament and degressive proportionality<br>→ Use models to predict future numbers of MEPs for potential new member states and analyze validity based upon parameters |
| Formative Assessments | Summative Assessments |
| → Desmos Graph Analysis activity<br>→ Mathematical interpretation of Article 14, Section 2<br>→ Different small group and whole class discussions<br>→ Gallery walk at the start of Day 2 to observe student thinking<br>→ Written explanations and accompanying Desmos graphs (when required) at the end of Days 1-4 | → Written proposal for model to use (pre- and post-Brexit), identification of any necessary parameters, predictions for inclusion of most recent countries granted candidate status (Ukraine & Moldova), and validity of those predictions at the end of Day 5<br>→ Unit Test (Day 6) |

## Learning Activities (1 week – 5 days): Lesson introduction, body, and closing

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| Day 1 | Class will open with the Graph Analysis Desmos activity (~30-35 minutes). This will allow students to review important vocabulary and equation identification for the different models we have studied (polynomial, rational, exponential, logarithmic), as well as introduce the idea of a logistic function, which will be explained in greater detail after completion of the activity. Students will also review key pieces of graphs (intercepts, discontinuities) and vocabulary associated with those.<br>- Teacher will monitor student progress through the activity with the Teacher Dashboard. Through the Dashboard, the teacher can see student responses and can follow up with students regarding any incorrect answers, either by moving around the room (Dashboard can be displayed on a laptop or iPad, allowing the teacher to be mobile) or through the chat feature in the platform.<br>- Though students are working independently on their own devices (school-issued Chromebook or personal device (not cell phone)), they can talk with one another about the questions and potential answers, as well as reasoning behind those answers. |

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| | Summarize Desmos activity, including clarifying any persisting misconceptions, and explain logistic model (~15-20 minutes).<br>- Answer student questions as well as issues observed frequently through the Desmos Dashboard during the completion of the activity<br>- Background on logistic model (what it is, what the parts mean): $y = \frac{1}{1 + ae^{-bx}}$<br>*Focus on the upper bound (number in numerator) – draw connections to horizontal asymptotes and maximum values<br>*Make connections back to base *e* and logarithms as well<br><br>Introduce European Union, with focus on European Parliament (~35-45 minutes)<br>- Brief history of the EU – use Slide 5 of the European Parliament 2022 PDF to help explain the growth since the creation of the European Coal and Steel Commission and the original 6 members<br>*Current configuration of the EU, including mention of Brexit – use the notes provided in the European Parliament 2022 Presentation with Notes PDF to quickly summarize<br>- Introduce European Parliament in particular detail<br>*Use Slide 1 from European Parliament 2022 PDF to show a picture of the European Parliament as introduction<br>*Show students the 'In the Past' tab to explain how the European Parliament has evolved as the only directly elected institution and select 'The Parliament and its Treaties' to see the list of treaties that have been adopted to govern the EU<br>+ Click on treaties that interest students<br>+ Highlight the Treaty of Lisbon, as it "enhances European Parliament's powers as a fully recognized co-legislator with increased budgetary powers [and] [a]lso gives Parliament a key role in the election of the European Commission President."<br>*Debrief with questions, observations, etc<br>*Connections with American governmental system – compare/contrast<br>+ Look specifically at House of Representatives, Electoral College<br>*Introduce terms *Minister of European Parliament (MEPs)* and *degressive proportionality*<br>+ "The more populous States agree to be under-represented in order to allow the less populous states to be represented better (pp. 11)"<br>+ Degressive proportional "does not provide us with one single solution, but instead offers an infinite number of options from which to choose (pp. 41)"<br>+ Excellent discussion in this paper about why simple proportionality is not sought (pp. 28)<br>+ Link with House of Representatives and numbers of reps being based on population, Senate independent of population<br>+ pp. 40 connects to Electoral College<br>*Reference Article 14, Section 2 of Treaty on European Union:<br>"*The European Parliament shall be composed of representatives of the Union's citizens. They shall not exceed seven hundred and fifty members, plus the President. Representation of citizens shall be degressively proportional, with a minimum threshold of six members per Member State. No Member state shall be allocated more than ninety-six seats.*"<br>+ Conclude class and connect everything reviewed and learned with the following question:<br>Explain how Article 14, Section 2 can be related to the different mathematical concepts and terms that we have reviewed today.<br>+ Students will have any time remaining in class to begin to formulate their responses<br>+ Student responses will be due at the start of class tomorrow via Google Classroom |
| Day 2 | Open class by placing students in groups of 2-3 and allowing them to share with one another their responses to the question posed at the end of class (~10 minutes)<br>- Teacher will move throughout the room at this time to listen to what is being shared (make note of analyses found interesting to be brought up later) – would expect to hear:<br>*Minimum and maximum with respect to 6 MEPs and 96 MEPs<br>*Maximum with respect to total of 751 MEPs<br>*Domain or range limited to $6 \leq \text{MEPs} \leq 96$<br>+ Link in later discussion exploration of whether MEPs is best described w/domain or range<br>*Discontinuities/asymptotes at 6 and 96<br>- Teacher will also want to listen for any misconceptions or important questions being posed in the groups<br><br>Teacher will reconvene the class to give observations from the different groups – can use this time to highlight anything heard during the group conversations (ie. specifically call upon certain groups to give their insights if they aren't initially volunteered) (~5-10 minutes) |

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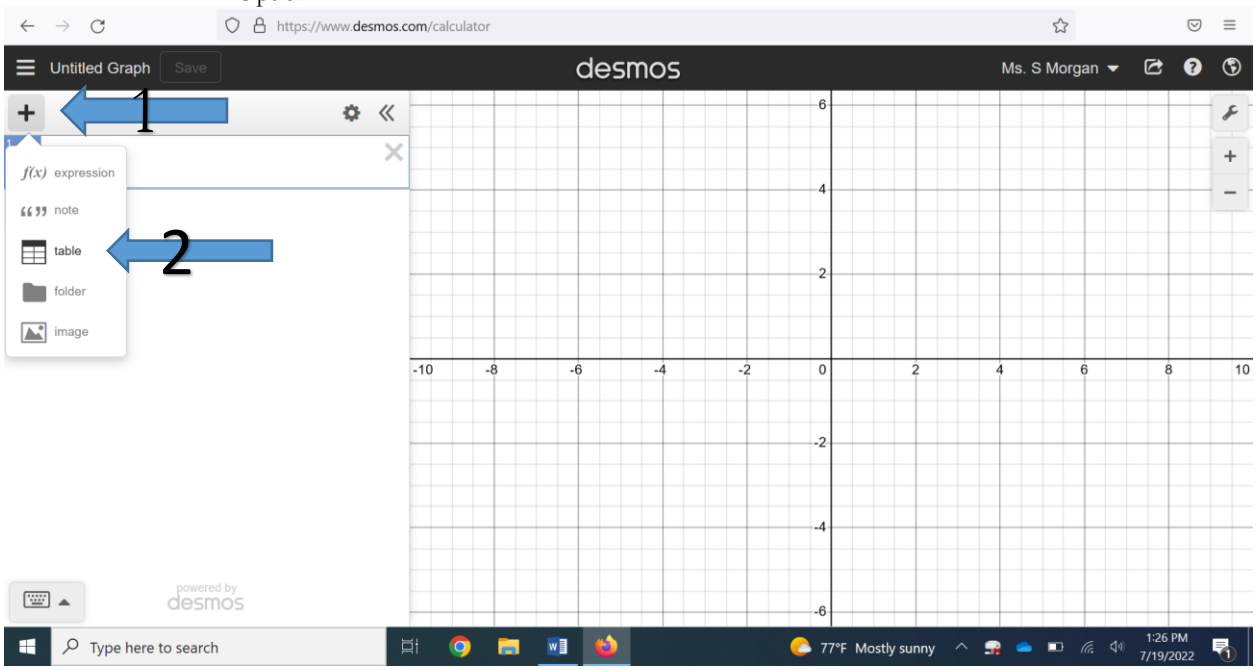
Teacher will pull up the infographic, but only the right side (post-Brexit)


- Make connection back to Day 1's conversation about Brexit, explaining that these are the MEPs after the United Kingdom left the EU (post-2020), with a total of 705 MEPs (~2 minutes)
- Remind students of *degressive proportionality* (~15 minutes)
    - *These numbers are based on the populations of each member state
    - *Connect this back to the discussion of domain and range:
        - + If MEPs are based on population, which is better suited for domain and which for range?
            - *Allow students to discuss with their groups the answer to this question (~5 minutes)
            - *As a class, have groups report out their thoughts
            - *As a whole group, decide that population is the independent variable, which is x, and MEPs the dependent variable, which is y, therefore MEPs are actually best represented in the range, and population is best represented in the domain.
- Question arises: What population data is used? (~5 minutes)
    - *QMV population data from The Composition of the European Parliament
        - + Briefly discuss qualified majority voting – 55% of Member States representing at least 65% of total EU population vote in favor when Council acts on proposal from Commission or High Representative; 72% of Member States representing at least 65% of total EU population vote in favor in other cases (New Council qualified majority Voting Rules In effect, 2014)

Now that we have population and MEPs, we can construct a table (~35 minutes)

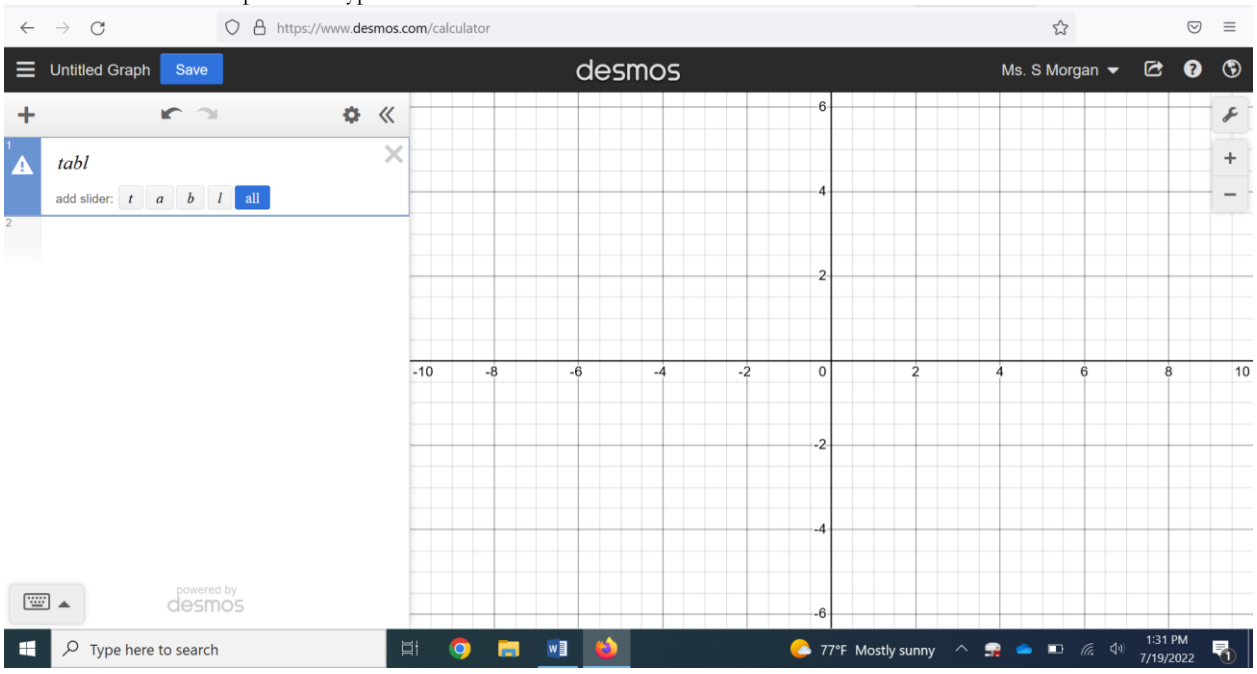
- Students will use Desmos.com on personal computers (school-issued laptop or own device) to construct a table (have done this in previous math courses, will review steps as needed) – 2 ways to do so:

+ Option 1:




Step 1: Click the '+' sign
Step 2: Click 'table'

+Option 2: Type the word 'table' into the row in Desmos




As soon as you complete the world 'table', Desmos will create a table for you

Regardless of the option chosen, a table will appear in the window:

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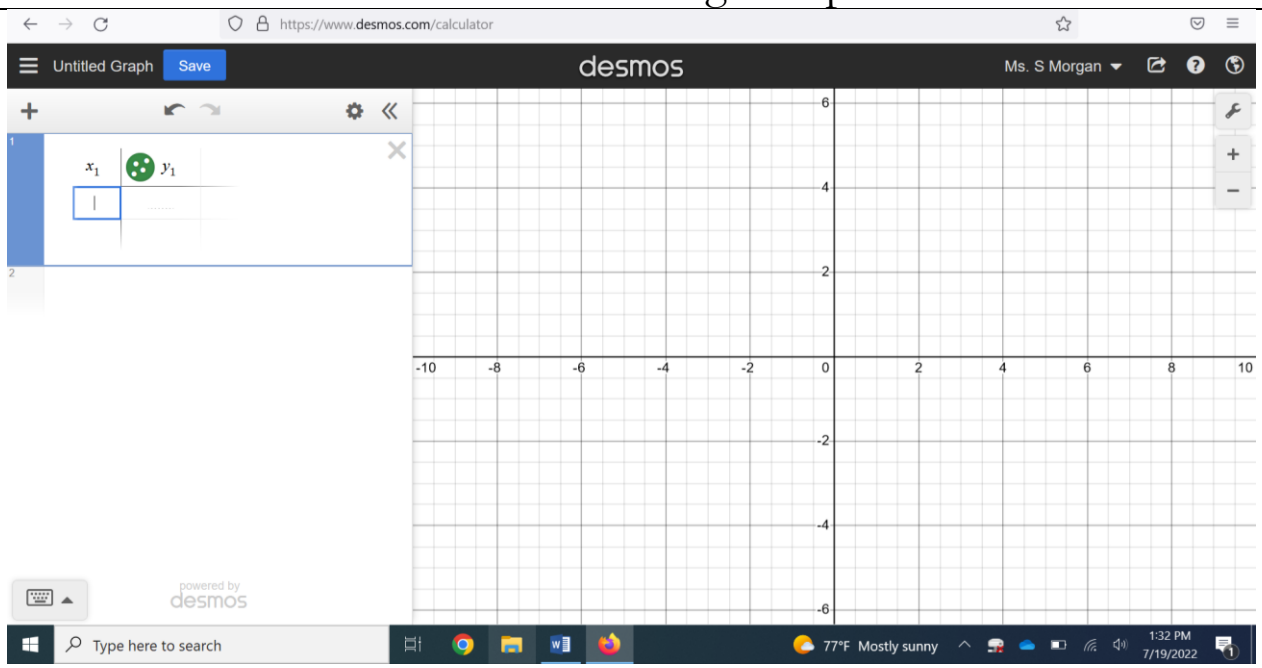


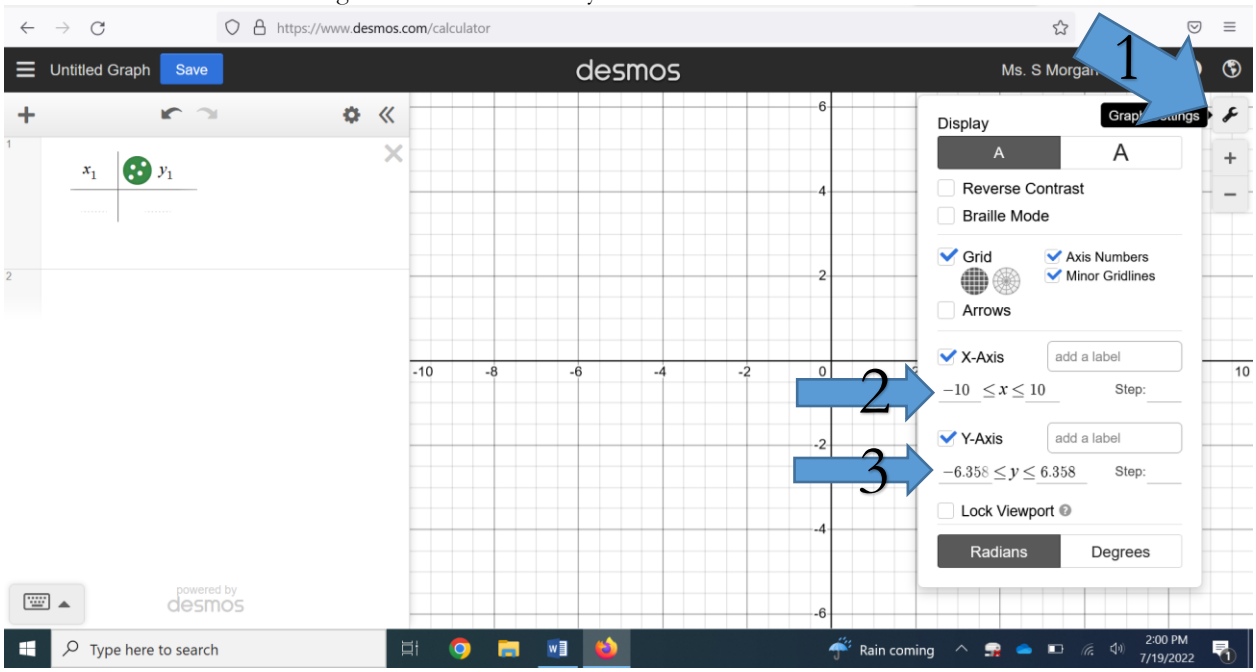

Question: What data do we input?

*Anticipate answers that population will go under $x_1$ and MEPs will go under $y_1$

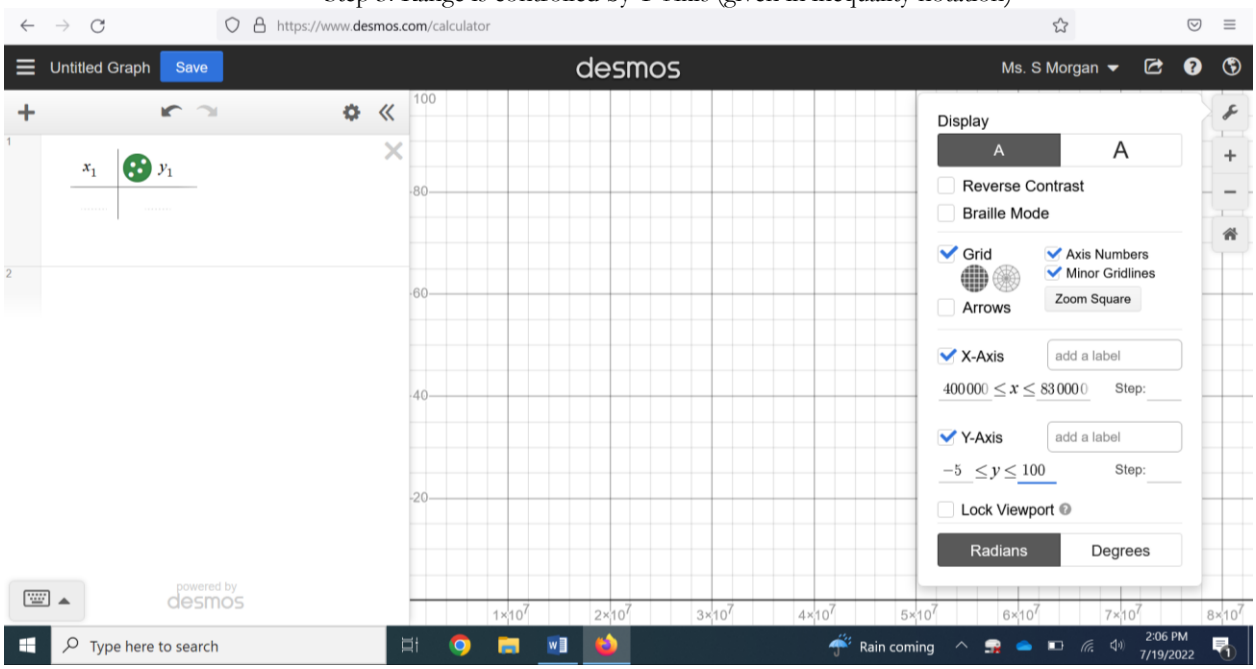
*Dig deeper by asking: Will you input those numbers – particularly population – exactly as you see them, either in the document or the infographic?

+ This may prompt more confusion
+ Consider asking this: Look at the window on your screen – will the data that we need to graph fit within this window? Give students ~1 minute to think about this (ask them to do this silently), then ask for thoughts/conclusions.
+ The goal is to make students think about how this window will need to change, and any changes made will need to be based on the way students elect to represent the numbers – do they want to type in the data as it is, or would they prefer to express it in small terms (such as in terms of millions, therefore 82,064,489 would be 82.064489)
+ Follow up question: If expressing in millions, should we round the decimal? Why or why not? If so, to what decimal place should we round? Have students turn and talk (groups of 2-3, depending on class size) to discuss these questions (~5 minutes)
+ Have students report out their thoughts – anticipate hearing:
  - Use decimal representation of population to keep y-values/range smaller, and therefore more manageable
    -Round to 2 or 3 decimal places for ease of data input
    -This provides excellent avenue to discuss precision in calculations and what each of those decimal places represents in terms of population (ie. 2 or 3 decimal places will still exclude hundreds if not thousands in the population – how will that impact our final result? What would that mean for the validity of our final model?)
  - Use population data exactly as provided to reduce error in the calculation
    -Students may reach this conclusion on their own or arrive at it through the conversation/questioning mentioned above
    -This is an idea that needs to be discussed explicitly so that students understand how precision and rounding can impact the validity of mathematical models
+ Allow students to come to consensus on how they want population data represented before moving on
  - If no consensus can be reached (students feel strongly one way or the other), allow different students to represent their data in different ways and analyze the models produced for differences or discrepancies
+ Ask students to change the window of their graphs on Desmos to best fit the data they will be inputting
  - Connect with discussion of domain and range (the x- and y-value selected will be a little outside the upper and lower bounds (maximums and minimums) of the domain and range to ensure the full

graph will be seen)
+ Change window in Desmos by:




Step 1: Click on the wrench icon (Graph Settings) in the top right corner of the screen
Step 2: Domain is controlled by X-Axis (given in inequality notation)
Step 3: Range is controlled by Y-Axis (given in inequality notation)

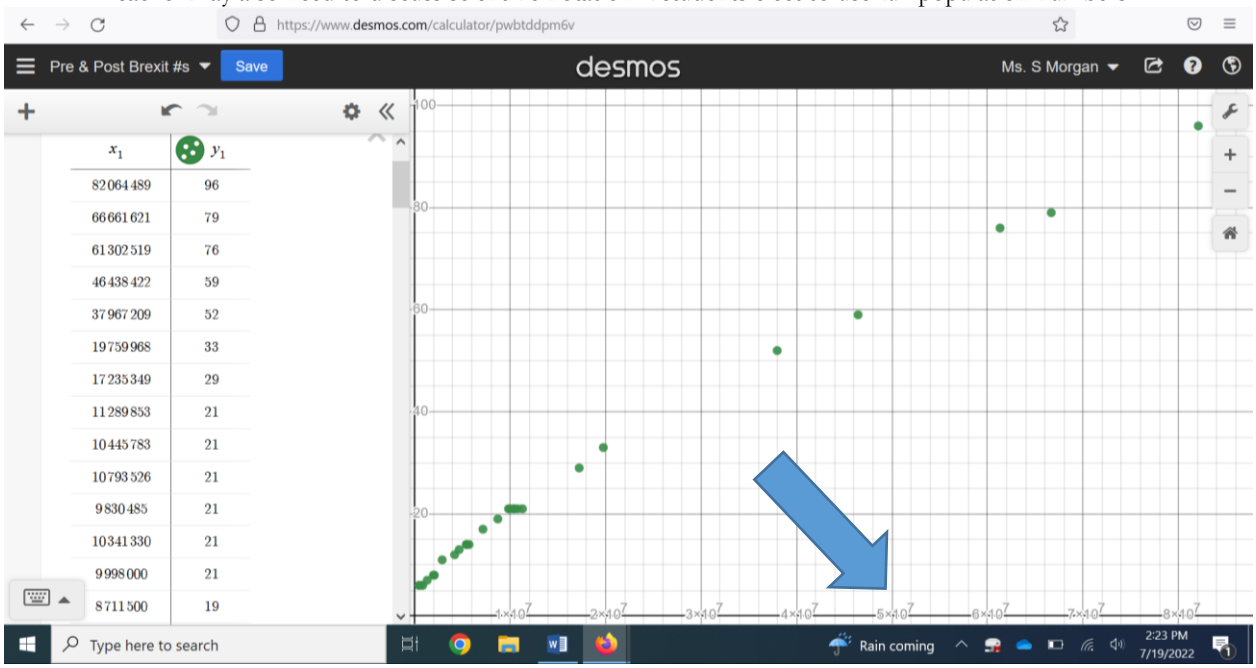



I have elected to express the population data as it is written, and subsequent tables used here will reflect that. If students choose to represent population data differently, their tables and windows will reflect that, but the overall shape of the graph produced by the table should be the same.

Have students input the data for the population and the post-Brexit MEP values (~10-15 minutes)

- This will take some time, and teachers should encourage students not to rush and to ensure they match up the correct country's population with the correct number of MEPs

- Teacher should move around the room and try to ensure numbers are matching up as well as answer any questions that arise
- Students may have question about their window, particularly if a point seems to be hidden behind the 'graph settings', 'zoom in', and/or 'zoom out' features in the top right corner of the screen – the teacher can help students readjust their window in order to make the point visible
- Teacher may also need to discuss scientific notation if students elect to use full population numbers:

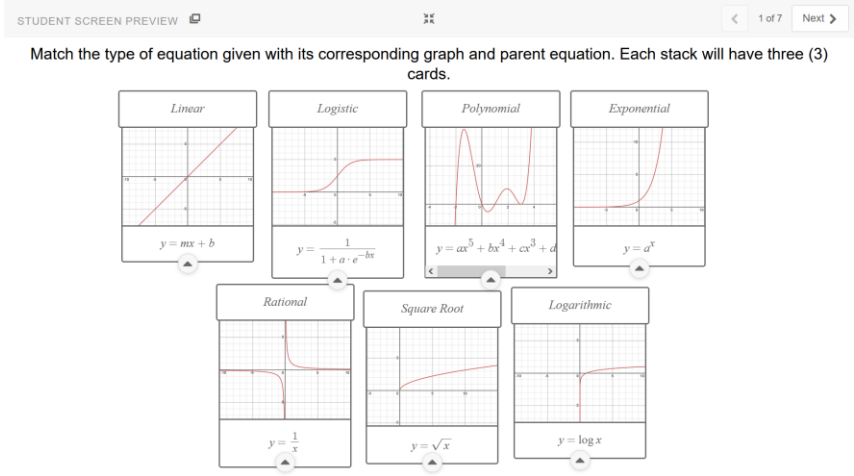



Quick review of scientific notation (~5 minutes):

- Whole number in front will always be $1 \leq x < 10$
- The factor of 10 attached will have an exponent that tells the direction to move the decimal and how far (ie. $5x10^7$ means 5 with the decimal moved 7 places to the right because it is a positive number and will therefore make the complete representation of the number bigger than 5, so $5x10^7 = 50{,}000{,}000$, whereas a negative exponent would move the decimal to the left)

Now that the table is input, have students save it (using the blue 'SAVE' button at the top left – will need to save through school email account) so that this table can be used in later classes.

Assignment: Review the different types of equations with which students are familiar from Day 1's Desmos activity:



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| | Ask students to consider which of these equations may best model the graph they've created and provide a written explanation that will be due at the start of class tomorrow via Google Classroom. Students are told that they may select more than one model if they believe both could be viable models. |
| Day 3 | Before class, the teacher will sketch these seven models on the whiteboard. When students come in to class, they will do a Gallery Walk to select the model(s) they thought best represented the data graphed on Day 2. Students will use a whiteboard marker to place a checkmark beside the model they selected (may place multiple checkmarks if selected multiple models) (~3 minutes).<br><br>The teacher will then ask students to group themselves based on the model they selected (if they selected more than one model, choose the one they feel best able to argue at this time). Depending on the size of the group(s), the teacher may need to break these groups down into smaller groups (3-4 students). Allow each group to discuss why they felt the model they selected is best.<br>- The teacher should move around the room to listen as groups are discussing<br>*Students have worked with scatter plots in previous math courses, so expect to hear discussions of lines of fit, a 'line' or 'graph' through the data 'touching' or 'coming close to' most or all of the points, etc (~5-10 minutes)<br>- Then, ask students to find one person in the room who selected a different model than them (ie. someone not in their current group) and discuss with them their reasons for selecting the different models (give ~2 minutes for shifting and regrouping – may need to allow groups of 3, depending on the numbers, then allow ~5-10 minutes for discussion)<br><br>Reconvene as a class and report out about what justifications were given for selecting a given model (~15 minutes)<br>- Many reasons will be conjecture – 'it looked like …' or 'it would be close to …' or 'I think …'<br>- These subjective rationales demonstrate the need for more objective and concrete mathematical reasoning and leads into the idea of mathematical regression<br>- Regression allows us to modify the format of the functions recognized in the Day 1's activity to let Desmos create lines of fit<br>- Examples:<br>*Linear: y = mx + b becomes $y_1 \sim mx_1 + b$ in Desmos<br>*Square root: y = $\sqrt{x} + b$ becomes $y_1 \sim \sqrt{x_1} + b$ in Desmos<br>*Logarithmic: y = a*ln(x) + b becomes $y_1 \sim a*\ln(x_1) + b$ in Desmos<br>*Rational: $y = \frac{1}{x}$ becomes $y_1 \sim \frac{a}{x_1}$ in Desmos<br>*Logistic: $y = \frac{1}{1+a*e^{-bx}}$ becomes $y_1 \sim \frac{1}{1+a*e^{-bx_1}}$ in Desmos<br>*Exponential: $y = a(b)^x$ becomes $y_1 \sim a(b)^{x_1}$ in Desmos<br><br>How do the equations need to change to do a regression? Why are these changes necessary?<br>- Ask students to think silently and independently about this question (~1 minute)<br>- Ask students to explain their thoughts and address their questions (~5 minutes)<br>*Looking for connections between the $x_1$ & $y_1$ in the new equations and the $x_1$ & $y_1$ headings on the table in Desmos<br>*Using the '~' symbol rather than '=' → students may or may not recognize this as a representation of approximation (since the model is a line of fit and may not pass through all points on the graph)<br><br>Ask students to open the Desmos graph they created the previous day on their computer (may take ~5 minutes for computers to load and students to log into the correct website; ~40 minutes to go through the entire discussion)<br>- Students will select one of the six models above and will type that equation into their Desmos graph<br>- For example purposes, the linear model ($y_1 \sim mx_1 + b$) is shown here: |

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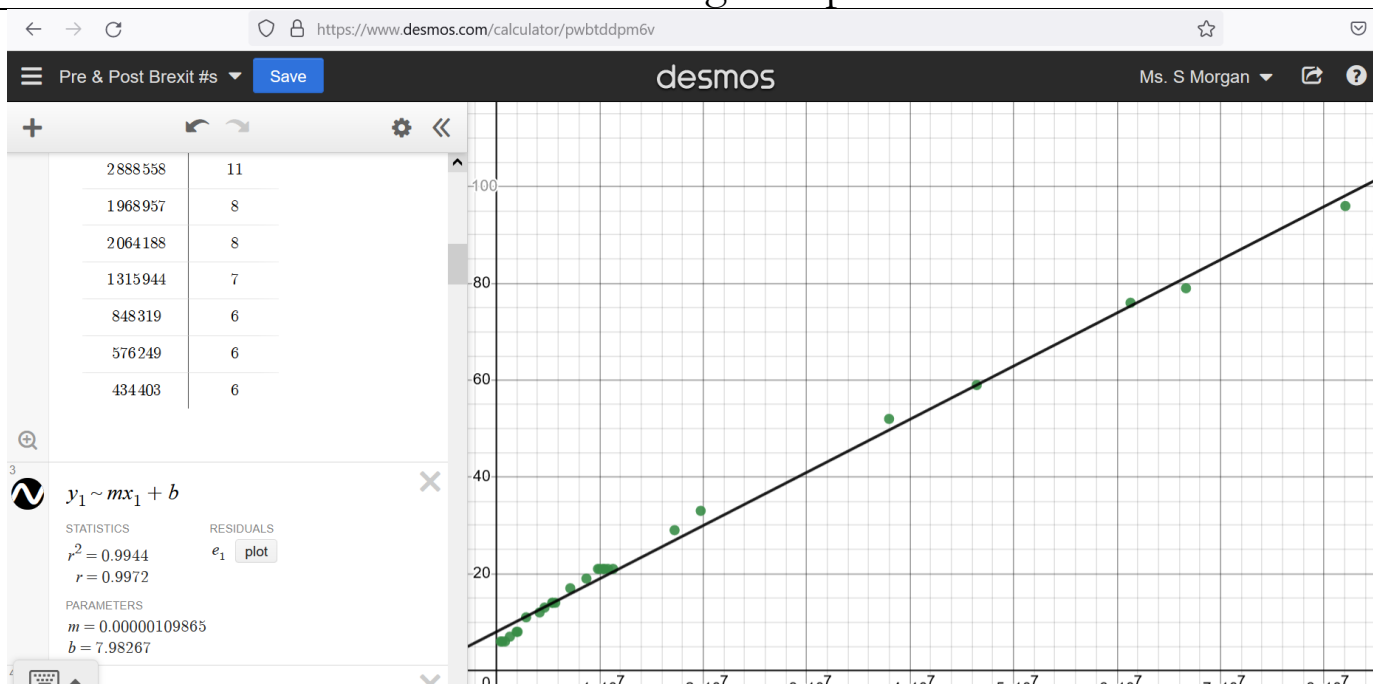


- Ask students to move to sit with at least one other person who is plotting the same regression equation as them (no more than 3 to a group) – if a student is the only person plotting a specific regression equation, that is okay as well (movement should take no more than ~2 minutes)
  *Students can compare what they see on their screens to ensure that they are doing things correctly
- Discuss the information given and the function plotted on the graph – the information below will be for the linear function, but other functions will produce values for their different parameters (*a*, *b*, etc); throughout the discussion, the teacher will need to move around the room to address questions as they arise, particularly from groups that did not select the linear model
  *The line represents the line of fit, which is the best predictor calculated based on the data provided
  *The information given below the regression equation (that which was typed into Desmos) yields a regression equation, or the equation for the line of fit – the teacher will need to explain the information here:
    +The $r^2$ value: The coefficient of determination → this number will be between 0 and 1, where values closer or equal to 1 will be strong fit (an $r^2$-value = 1 represents a perfect fit) and value closer to or equal to 0 will be a poor fit
      -This indicates the possibility for correlation, ***not*** direct causation
      -The $r^2$ value for this function is .9944, which indicates a very strong fit
    + The values under the 'Parameters' heading: for a linear function, *m* represents the rate of change (slope) and *b* represents the y-intercept, and these values would be filled into the generic equation typed into Desmos to produce the regression equation: $y = (1.09865 \times 10^{-6})x + 7.98267$
      -For *m*, use the review of scientific notation to clean up the number a bit: $m = 1.09865 \times 10^{-6}$
      -For *b*, 7.98267 is a value worth exploring
        *Ask students: What does the y-intercept represent? Possible answers include:
          -The ordered pair (0, 7.98267)
          -The starting or initial value (generic answer from a previous math course)
          -The number of MEPs when the population is 0 (the more specific explanation of the previous bullet)
      -Ask students: Does it make sense to have almost 8 MEPs for a population of 0 people?
        *The answer should be 'no' → this will lead to a follow up question: How does this impact the domain and range of this function? And how does that domain and range compare to the regression equation graphed on the screen?
          -The regression equation on the screen is a continuous function that, in theory, has a domain and range of All Real Numbers → does that make sense for our scenario?
          -No, and this allows us to reconnect to the opening activity of Day 2: There are restrictions on our data based on the real-world scenario it describes – our data is only 'valid' within the domain of the

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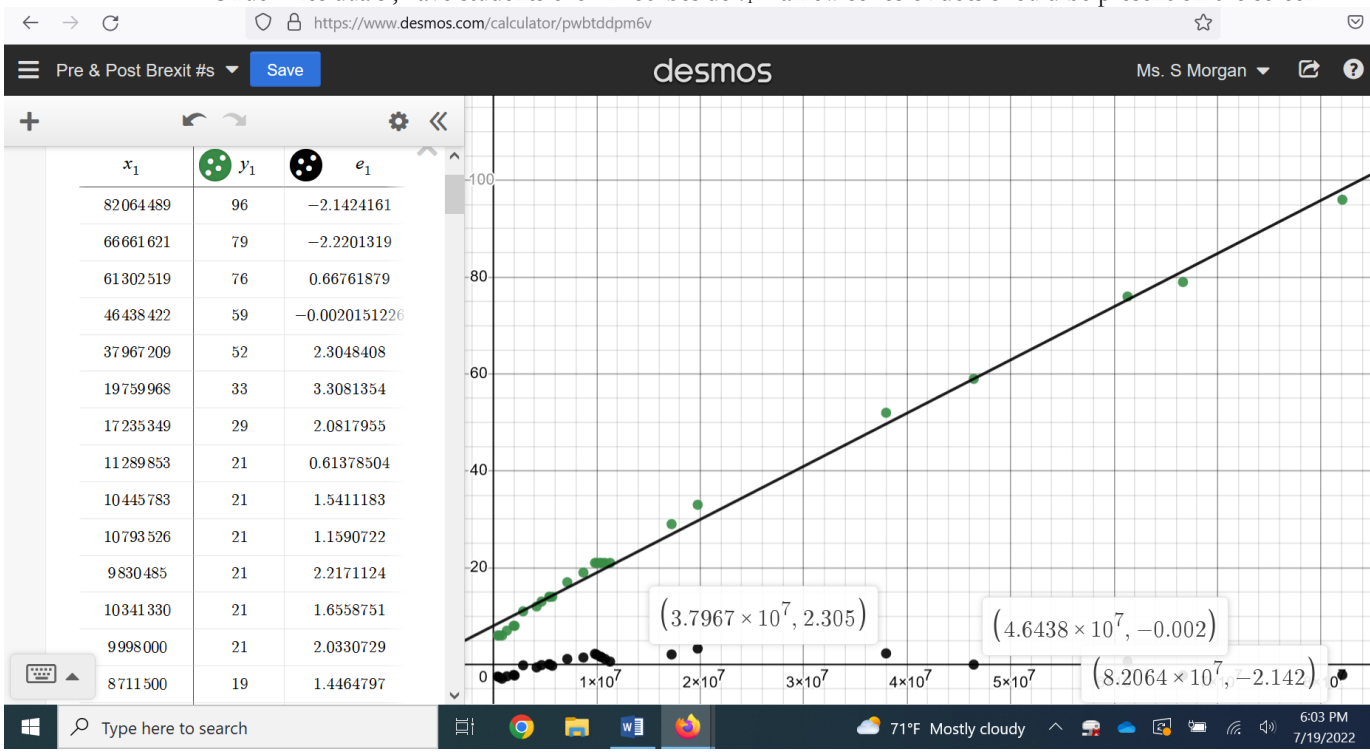
| | populations we used in our data set, not for a population of 0, negative x-values, or value larger than those in our data set<br>-Similar discussions about the range – our data should only exist between values of 6 MEPs and 96 MEPs<br>+Under 'Residuals', have students click 'Plot' beside $e_1$ – a new series of dots should be present on the screen<br>  <br>-The teacher should point out to students that these points move around (above, below, on) the x-axis<br>-Residuals tells us how far off the line of fit the data point is – the three that are selected as examples show that:<br>*The population point 3.7967 x $10^7$ (37,967,209 in the table) is a distance of 2.305 above the line<br>*The population point 4.6438 x $10^7$ (46,438,422 in the table) is a distance of .002 below the line (such a small distance shows that the line essentially passes through the point)<br>*The population point 8.2064 x $10^7$ (82,064,489 in the table) is a distance of 2.142 below the line<br>-For a better fit, should you see the residuals closer to the x-axis or further away from the x-axis? Why?<br>*Ask students to take 1 minute to think silently, then report out their thoughts<br>*Ultimately want students to conclude that they want residuals closer to the x-axis because that means the distance between the line of fit and the actual data point is small<br>- Ask students to save their graph (click 'Save' on the blue square at the top) to return to tomorrow<br><br>Assignment for the day (due at the start of class tomorrow via Google Classroom): Consider the regression equation you graphed. Explain your thoughts on the validity of your model. Include any limitations or parameters that may be needed to justify the appropriateness of your model. Do you think another model may work as well or better? How will you determine this? |
| Day 4 | To open class, ask students to find someone who used their same model yesterday (if possible, not the same person you partnered with yesterday) – groups should stay at 2-3 (if a student was the only one to select a specific model, have them join with a group of 2 for this discussion)<br>- Students should discuss their responses to yesterday's assignment (~5-10 minutes)<br>- Teacher should circulate throughout the room to hear rationales and listen for understanding and misconceptions<br>- Teacher should then reconvene to allow students to debrief and clarify any misconceptions (~5-10 minutes) |

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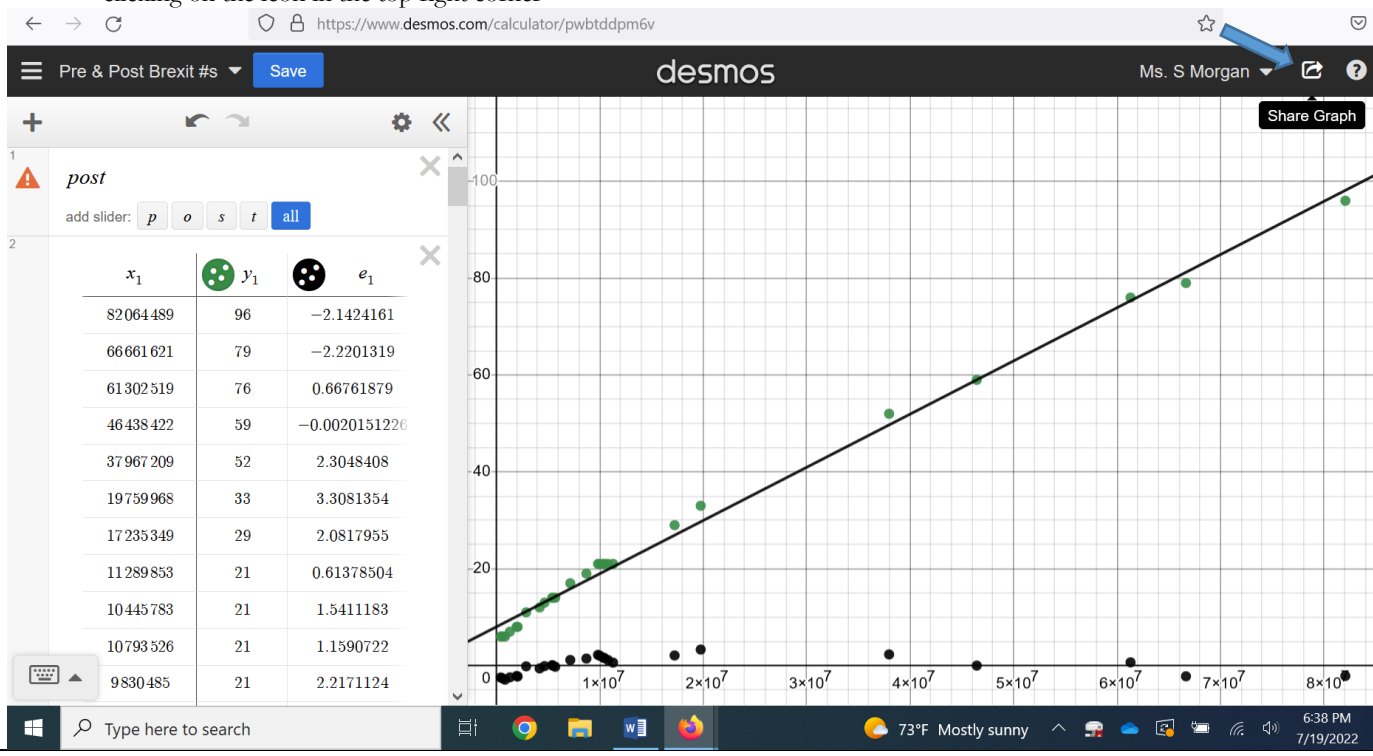
The teacher should ask: What is the only model for which we haven't looked at a regression equation?

- Answer: Polynomials
- The teacher will then explain that this is because we have many types polynomials, and the equation must change slightly as the polynomial gets bigger (~20 minutes) – Examples:
  *Quadratics: $y = ax^2 + bx + c$ becomes $y_1 \sim ax_1^2 + bx_1 + c$
  *Cubics: $y = ax^3 + bx^2 + cx + d$ becomes $y_1 \sim ax_1^3 + bx_1^2 + cx_1 + d$
  *Quartics: $y = ax^4 + bx^3 + cx^2 + dx + f$ becomes $y_1 \sim ax_1^4 + bx_1^3 + cx_1^2 + dx_1 + f$
  *Quintics: $y = ax^5 + bx^4 + cx^3 + dx^2 + fx + g$ becomes $y_1 \sim ax_1^5 + bx_1^4 + cx_1^3 + dx_1^2 + fx_1 + g$
  *Any subsequent polynomials can be built by simply adding additional parameters on to the end of the function
- Question (either from the students or from the teacher if students don't take note): Why do we not use an $e$?
  *Think back to yesterday's discussion of residuals – what variable was used by Desmos for the residual?
  *Answer: $e$ → therefore, we do not use it as a parameter when constructing a polynomial regression equation

Teacher will task each student with selecting at least 3 additional models (beyond the one that they've already done) that they think would have potential to be the best model to use (will end with at least 4 total)

- In Desmos, students will need to analyze each of the additional models they are testing – these analyses should include:
  *examination of the coefficient of determination
  *examination of the residuals
  +Students will need to be shown how to erase residuals from each test → click to the right of the '1' in '$e_1$' in the table, then Backspace twice to remove the residuals from the graph
  *discussion of any restrictions using terminology appropriate to Math 3 (minimum, maximum, domain, range, discontinuity, etc)
- Students will need to be shown how to 'turn off' a function so that it does not complicate what they are viewing in the screen – they can do this by clicking on the colored circle to the left of where they typed the function until it becomes opaque; they can 'turn on' the function by clicking the opaque circle until it becomes color-filled again
- Students will need to be shown the logistic regression function, as this is technically a Math 4 calculation: $y_1 \sim \frac{maximum}{1+a*e^{-bx_1}}$; students will need to input the maximum number allowed in the numerator
- All analyses should have a written explanation (either via Google Doc, Microsoft Word, or on paper and then photographed to upload into Google Classroom)
- Students will be instructed to save their analyses in Demsos, then share the graph Google Classroom via the link found by clicking on the icon in the top right corner



- The teacher should circulate through the room while students are working to answer any questions or help technical issues
- Any student who finishes early should be directed to the following websites dealing with EU Enlargement:
  *Candidate Countries and Potential Candidates and read through the tabs for:
    +Introduction
    +Main Challenges
    +Candidate Countries and Potential Candidates
    +Regional Cooperation
    +Civil Society
    +Financing
  *It is not anticipated that any student will have time to read through all of these tabs, but this will be a good introduction to what is coming next

Using the aforementioned tabs and the European Union website on Becoming part of the European Union (about halfway down the page), the class will examine what it takes to be considered for candidate status to the EU and potential MEPs (~20 minutes)

- Current candidates include Albania, Montenegro, Serbia, North Macedonia, & Turkey (from website) as well as Ukraine & Moldova (as of June 2022)
- Discuss the criteria that would be necessary (using the tabs) as well as the time necessary that it can take for countries to be granted membership (Turkey, for instance, has been a candidate since 1999)
- Consider the populations of the countries the first 5 countries on that list (data found by Google search):
  *Albania – 2.838 million (as of 2020)
  *Montenegro – 621,718 (as of 2020)
  *Serbia – 6.908 million (as of 2020)
  *North Macedonia – 2.083 million (as of 2020)
  *Turkey – 84.34 million (as of 2020)
- Consider this against the data that we have from current EU members and their populations → how would these candidates compare?
  *Recognize that Turkey would become the most populous nation, Montenegro would be of the smallest members, and the other three would be on the smaller end as well
- Can we predict how many MEPs Turkey would have, based on our models?
  *The teacher should ask students to select what they believe is their best model and have it 'turned on' on their graph, while turning off' all other models
  *The teacher should ask: What variable represents population? Students should answer the x-value
  *From there, the teacher will explain that this means that Turkey's population is represented on the graph with x = #
    +This number will need to be expressed in the same way as the data students have used in their tables – in the example included in this lesson, this would mean changing 84.34 million into x = 84,340,000 in order to be appropriate for the model
  *The teacher should ask students to type x = # (whatever numeric representation fits their table) into Desmos
  *As soon as the equation is typed into Desmos, it will yield a vertical line. Students will observe that this line intersects the line of fit. They will click (or be told to click, if they do not do so independently) on the point of intersection of these two lines. When they click on the point of intersection, Desmos will provide the ordered pair for the point.

# Brussels-Lux Study Tour 2022
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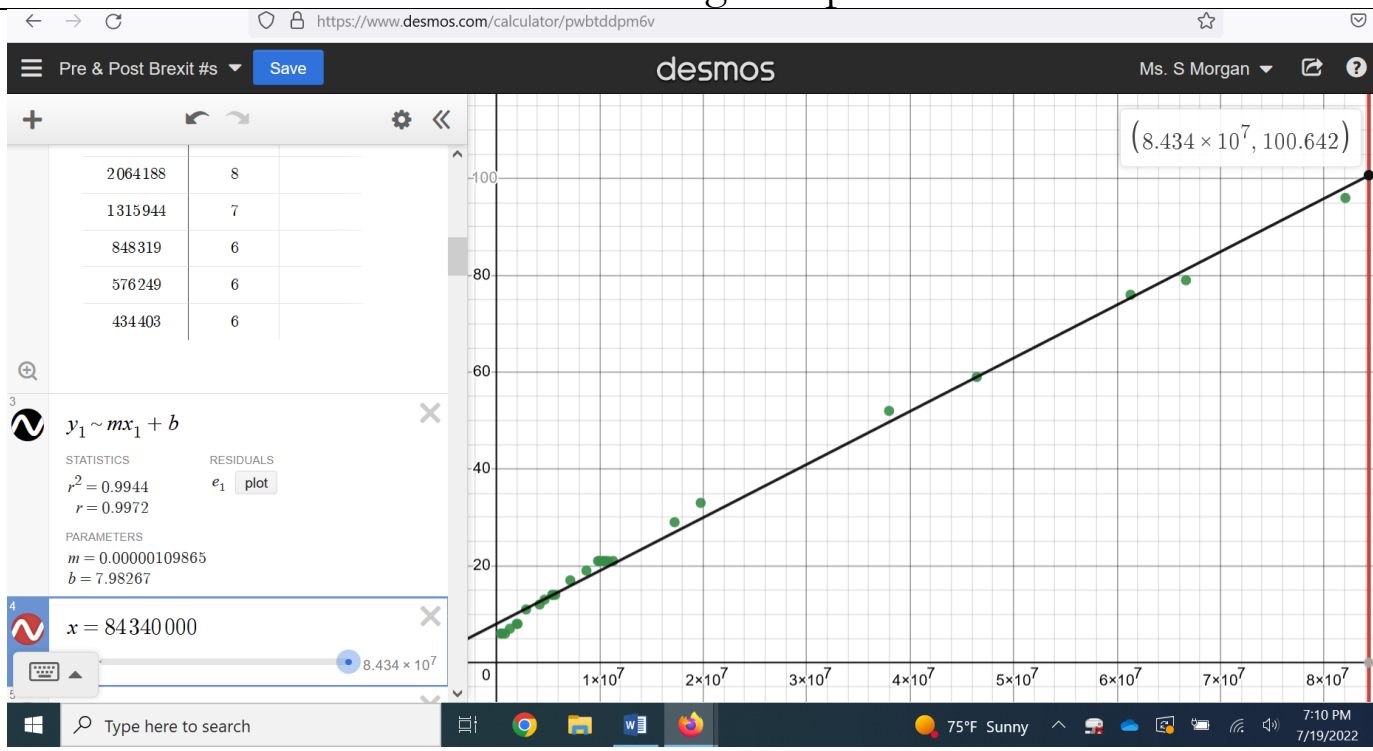


*What does this tell us about the number of MEPs for Turkey predicted by our model?

+Students will recognize that the x-value is the population, so the y-value is the number of MEPs – therefore, the number of MEPs precited by a linear model is 100.642, or ~101

*The teacher will ask: Does it make sense that a country could have 101 MEPs?

+No, for at least two reasons:

1. The number of MEPs for any member state cannot exceed 96 (per Article 14 Section 2 of Treaty on European Union)
2. This would make the number of MEPs in the European Parliament exceed the 751 total members allowed by Article 14 Section 2 of the Treaty on European Union

*What would this mean for the validity of the linear model?

+Possibly that it isn't good at modeling the potential MEPs for larger member states

+As Germany currently has the maximum number of MEPs allowed, perhaps the inclusion of Turkey into the EU would force a reshuffle of MEPs for other member states

Assignment: Use your model to predict the number of MEPs for Albania, Serbia, Montenegro, and North Macedonia. Determine if these numbers of MEPs make sense based on what we know of the European Parliament and mathematically. Submit your answers and a link to your Desmos graph via Google Classroom before class tomorrow.

**Day 5**

To open class, have students group with others who have been using their same model (~3-5 per group, though smaller is okay) to discuss the number of MEPs they predicted for Albania, Serbia, Montenegro, and North Macedonia and their conclusions as to the validity of these predictions. (~3-5 minutes)

- The teacher should move throughout the room during these discussions
- The teacher should take note if any student has moved from one model group to another and ask if there was a reason behind the selection of a different model

Reconvene the class to debrief about the different models and if anyone changed their thoughts on the best model to use (~5-10 minutes)

Return to the scenario regarding Turkey from yesterday – Can the number of MEPs be 'reshuffled' and change (either increase or decrease) (~15 minutes)?

- The answer is 'yes' → Consider Brexit by pulling up the full infographic

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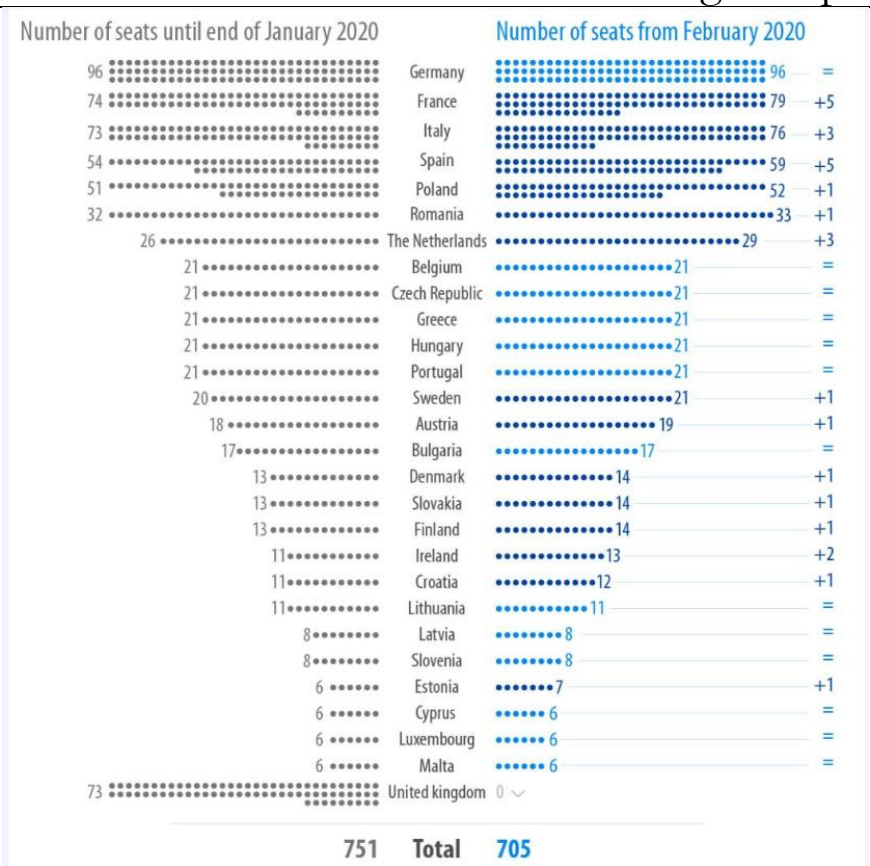


- The teacher should ask: What do you notice about the Pre- and Post-Brexit composition of the European Parliament and the allocation of MEP's? Some observations may include:
  *There were 751 MEPs, which is as many as is allowed
  *The UK had 73 MEPs
  *14 countries gained MEPs as a result of the UK leaving the EU
  *Only 27 of the UK's 73 MEPs were redistributed to other countries
  *The UK was one of the 4 largest EU countries while it was a member, so if its withdrawal from the EU caused a reshuffle, it makes sense that the inclusion of a larger nation would also cause a reshuffle, only this one may involve some countries losing MEPs

Now, let's mathematically consider Brexit (~40 minutes)

- Students should be instructed to open their Desmos graph and add a new table at the bottom including the values for pre-Brexit (UK population value taken from Composition of European Parliament document – UK is boxed in blue in figure below)
- Students should be instructed to take their time inputting the values to ensure that they are all done correctly
- Students can insert the UK values wherever they choose in the table (location doesn't matter), so long as the correct population value aligns with the correct number of MEPs

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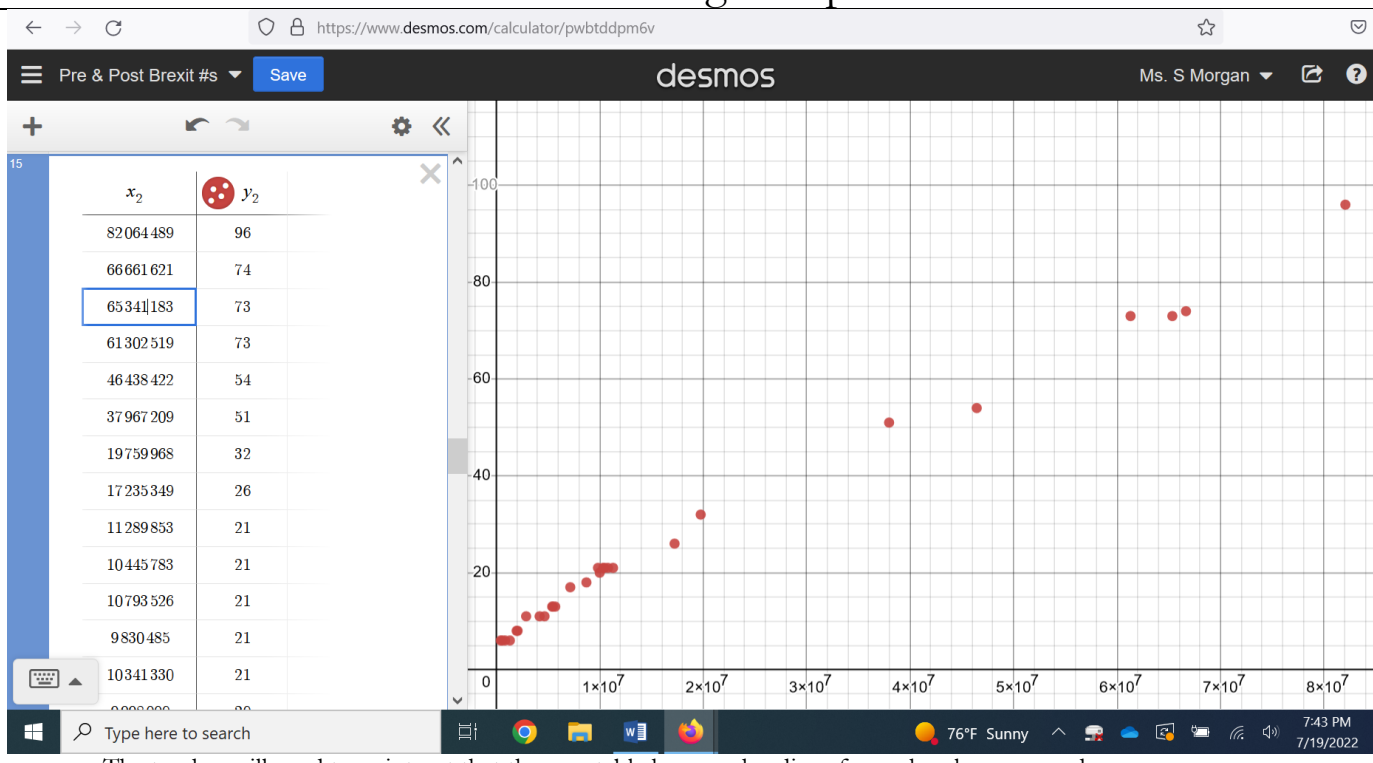


- The teacher will need to point out that the new table has new headings for each column: $x_2$ and $y_2$
- The teacher should ask: How does this impact the regression models you will use?
  *The structure of the equations will not change, but the subscripts used will need to change in order to pull data from the correct table (ie. a linear model for the Pre-Brexit data would be $y_2 \sim mx_2 + b$)
- The teacher will ask students to consider the 4 models they analyzed yesterday to determine the best model for Post-Brexit data
  *Analyze the same models with Pre-Brexit data - as before, these analyses should include:
    +examination of the coefficient of determination
    +examination of the residuals
    +discussion of any restrictions using terminology appropriate to Math 3 (minimum, maximum, domain, range, discontinuity, etc)
  *The teacher may need to review how to 'turn off' and 'turn on' different functions as well as how to delete residuals from the table (can be done as a class or on an individual basis, depending on what the teacher determines the class needs)
  *An additional element will be included in this analysis – students ***must*** document if there are any changes in the coefficient of determination, any major changes in residuals, and/or any changes in restrictions
- Any student who finishes early should start reading this press release: Grant EU candidate status to Ukraine and Moldova without delay, MEPS demand

Two most recent countries to gain candidate status are Ukraine and Moldova (use the article above to introduce and explain – population totals come from Google search) (~5 minutes)

- Moldova – 2.618 million (as of 2020)
- Ukraine – 44.13 million (as of 2020)

Final assignment: You must write a proposal for the equation to use to model the number of MEPs if:

1. Moldova joins the EU
2. Ukraine joins the EU
3. Both countries join the EU

You may use any model you wish – based on Pre- or Post-Brexit data – but you must justify your proposal discussing the different aspects of graph analysis that we have looked at over the last 5 days as well as your prediction for how many MEPs Moldova or Ukraine or both would be allotted in the European Parliament, ensuring that your proposal meets all the requirements of Article 14

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|---|---|
| | Section 2 of the Treaty on European Union. If, at any point, it is impossible for one or both nations to join the EU with the predicted number of MEPs, explain why this is the case.<br><br>Proposals, along with links to your final Desmos graph, must be submitted in Google Classroom before class starts tomorrow. |
| Day 6 | Spend ~15-20 minutes discussing the different models students proposed and their rationales.<br>- Undoubtedly, a student will ask what the 'correct' answer is<br>- "Scholars have noted that 'there is neither a formula to determine the vote weight of each State on the Council of the EU nor a formula to calculate the number of seats in Parliament' (pp. 23-24, Composition of the European Parliament)"<br>- There is not actually one set formula – there are statistical calculations that can be performed, but these are on a case by case basis that revolve around the population(s) of the country(ies) to be admitted (pp. 27-24, Composition of the European Parliament)<br><br>Spend ~10 minutes addressing any final questions<br><br>Students will use the rest of class to take their Unit Test |

**Resources and Materials**

Desmos Activity – Graph Analysis (Intro Day 1)

Desmos online graphing calculator

European Parliament 2022 Presentation PDF

European Parliament 2022 Presentation with Notes PDF

The Composition of the European Parliament

Directorate General for Internal Policies of the Union. (2017). *The composition of the European parliament*, Policy Department for Citizens' Rights and Constitutional Affairs. https://www.europarl.europa.eu/RegData/etudes/IDAN/2017/583117/IPOL_IDA%282017%29583117_EN.pdf

Infographic: How many seats does each country get in the European Parliament?

European Parliament. (2020). *Infographic: How many seats does each country get in the European Parliament?* News: European Parliament. https://www.europarl.europa.eu/news/en/headlines/eu-affairs/20180126STO94114/infographic-how-many-seats-does-each-country-get-in-in-the-european-parliament

Treaty on European Union, Article 14 Section 2

No author. (n.d.). *Consolidated version of the Treaty on European Union, Title III – Provisions on the Institutions, Article 14*. Eur-Lex: Access to European Union Law. https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=celex%3A12016M014

New Council qualified majority Voting Rules In effect

No author. (2014). *New Council qualified majority Voting Rules In effect,* Council of the European Union and the European Council. https://www.consilium.europa.eu/en/documents-publications/library/library-blog/posts/new-council-qualified-majority-voting-rules-in-effect/

Grant EU candidate status to Ukraine and Moldova without delay, MEPS demand

No author. (2022). Grant EU candidate status to Ukraine and Moldova without delay, MEPs demand. *News – European Parliament.* https://www.europarl.europa.eu/news/en/press-room/20220616IPR33216/grant-eu-candidate-status-to-ukraine-and-moldova-without-delay-meps-demand

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MEET EU

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EUROPEAN UNION