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TITLE: A GENERAL-EQUILIBRIUM THEORY OF
PARALLEL MARKETS IN CENTRALLY-
PLANNED ECONOMIES

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## Contents

Executive Summary 111

Introduction 1

Part I. The Parallel Market: A Static Analysis 3
   1. An Initial Equilibrium 3
   2. A Simple Parallel Market Model 6
   3. Penalties on "Resource Seeking" 14
   4. Price Controls and Quantitative Allocations 17
   5. Summary 23

Part II. The Parallel Market: A Dynamic Analysis 25
   1. A Two-period Model 25
   2. A CPE Model 30
   3. The Parallel Market when Private Producers are More Efficient than Bureaucratically Run State Enterprises 32
   4. The Parallel Market with Two Consumption Goods 38
   5. Summary 44

Part III. A Computable General Equilibrium Model 46
   1. The Input-Output Table 46
   2. The Model 47
   3. Functional Forms 47
   4. The Benchmark Equilibrium 48
   5. Results 49
   6. Summary 51
A General-Equilibrium Theory of Parallel Markets in Centrally-Planned Economies

Executive Summary

Based on the accounts provided by experts, it can be safely asserted that the "parallel" market has become an integral part of many centrally planned economies (CPEs). Given this fact, we cannot properly understand the legal, "first" economy without studying its relationship with the parallel or "second" economy. For this reason, in the present report, I provide a detailed general equilibrium analysis of CPEs which take explicit account of the parallel market.

A major contribution of the report is the construction and implementation of a simulation model of the Soviet economy incorporating the parallel market. Under the conservative assumptions that the illegal economy is 4 percent of the official economy and that private firms are no more efficient than state enterprises, the model predicts that if the parallel market is eliminated, the capital stock will have to be raised by 2 percent in order to maintain the existing living standards. The required increase in capital stock will be much larger if we assume that the parallel economy is larger, say 10 percent of the official economy, and that private firms are more efficient than state enterprises. The model also predicts that the parallel market leads to a reallocation of resources in favor of present consumption. Simulations of the model show that over time, the parallel market is likely to grow, especially if enforcement of laws against offenders gets weaker. Tables 3-5 at the end of the report provide detailed results of various simulations.

The theory in the report is based on two central themes in the literature on CPEs. First, I formalize the idea that private entrepreneurs can operate technically more efficiently than bureaucratically run state enterprises. Second, I analyze the implications of the parallel market activity resulting from price controls and quantitative allocations.
The report constructs a series of general equilibrium models to explore the links between the first and second economies. My emphasis is on looking at different ways in which illegal economic activities can influence the first economy rather than generating a specific set of results.

The report is divided into three parts. Part I develops the theory of parallel markets in a static framework while Part II is based on a dynamic model. In Part III, I simulate the Soviet economy using a computable general equilibrium model.

1. Static Analysis

An important objective of my analysis is to capture the most important links between the first and second economies without losing either tractability or internal consistency. Attainment of this objective requires a considerable abstraction from reality as illustrated by the schematic flow chart of the economy in Figure I. The figure represents the economy as hypothesized in Part I of the report.

As noted earlier, the analysis in Part I is static in nature. I postulate a CPE which produces two goods using two inputs, labor and capital. One good employs more labor per unit of capital than the other and is referred to as the labor intensive good. The other good is called the capital intensive good. Capital is owned by the state. All official allocation decisions are made by the state's planning agency. The latter uses appropriate shadow prices as signals to implement its optimally chosen plan.

It is assumed that there are two types of agents in the economy, workers and bureaucrats. The latter have a strong preference for the capital intensive good relative to the former. Workers receive wages from enterprises while bureaucrats are paid salaries out of state revenues. In reality, a large part of the state's revenues is spent on capital formation. This feature is incorporated in the dynamic model of Part II. As static models cannot allow for investment, the best compromise is to assume that all
revenues are given to bureaucrats. We can, of course, think of the latter's strong preference for the capital intensive good as representing the demand for investment.

An obvious question concerns the source of inputs for producers in the second economy. As the total endowments of capital and labor in the economy are fixed, the second economy must obtain its inputs from the first economy. Thus, I assume that enterprises sell a part of their capital to second economy producers for a profit. Analogously, workers divert a part of their services to the second economy in the hope of getting a higher wage. There is a good deal of support for these assumptions in the empirical literature.

A final notable feature of the analysis is the explicit consideration of the state's enforcement policy. The state monitors illegal producers. This fact means that the second economy producers face an uncertain environment. Taking the probability of detection and punishment (involving complete confiscation of goods) as given, private producers maximize expected profits. A separate section of Part I considers the case when the state monitors the resource diversion activity of enterprises.

The analysis and conclusions in Part I can be understood most easily by assuming that the parallel market exists in only one sector, say the labor intensive sector. Extensions to the case when both goods are subject to illegal production are straightforward and can be found in the report.

Assume that the state implements a plan which is characterized by a larger production of the capital intensive good than that needed to ensure demand-supply balance. At the official price, there will be a shortage of the labor intensive good and the state will have to resort to quantitative allocations. The shortage implies that illegal production is profitable as long as resources can be obtained at prices that are not much higher than those paid by enterprises. It turns out that if the state takes the parallel market into account in choosing its plan, parallel production does not alter
the overall allocation of resources between the two sectors. Moreover, if production costs are the same in the official and second economies, total outputs of the two goods remain unchanged as well. All that happens is that production of the labor intensive good moves partially underground and the good is sold at a higher price in the second than in the first economy. Of course, if illegal firms are more efficient as they are likely to be, the total availability of the good will increase which is potentially beneficial.

2. Dynamic Analysis

In Part II, I develop a two-period model. This model enables us to incorporate the important institutional feature that the state uses the income generated by capital for investment.

As the two-period model is more complicated, I simplify the analysis by assuming that there is only one consumption good. The other good is a pure investment good. A further simplification is that workers and bureaucrats have the same tastes. After investment needs have been met and bureaucrats' salaries paid, any remaining revenues are redistributed among workers.

The state chooses its plan to maximize a social welfare function characterized by relatively more future consumption than that desired by individuals. This leads to a loss of utility as measured by individual preferences. If there are private firms that are more efficient than state enterprises, the parallel market can emerge. It is shown that if the state chooses the optimal plan taking the resources in the official economy as given, emergence of the parallel market lowers investment and raises both current and future consumption, with the former rising proportionately more. Welfare based on private preferences necessarily rises.

The model is extended, after certain simplifications, to allow for two consumption goods and one investment good. In this extension, the parallel market emerges due to a shortage created by relative prices that fail to clear the markets. This extension also incorporates bribes into the analysis. The
model is employed in Part III to simulate the Soviet economy. The results of various simulations have been summarized in the second paragraph of this summary.

Interested readers may obtain machine-readable copies of the model (programs for use with GAMS software) and data, with documentation, from the National Council for Soviet and East European Research.
Figure I: A Schematic Flow Chart of First and Second Economies.
The analysis of the "parallel" or "second" economy has become a matter of urgency in view of the increasing recognition of its importance in many economies. The phenomenon was widely noted and discussed in the context of the extensive control-generated transactions in developing countries, as in India (e.g., Bhagwati and Desai, 1970). It has recently been the subject of a growing empirical literature in the context of centrally planned economies (CPEs), with notable contributions by Simes (1975), Grossman (1977) and Katsenelinboigen (1977) on the importance of the parallel economy in the Soviet Union.

Although an important theoretical literature exists incorporating parallel markets into conventional general-equilibrium analysis of market economies in an essential fashion (Bhagwati and Hansen, 1973; Sheikh, 1974; Pitt, 1981; and Martin and Panagariya, 1984;), the analysis does not reflect any of the key aspects of CPEs. Addressing the issue in the context of CPEs, however, Wellisz and Findlay (1986) have recently undertaken a pioneering parallel-market analysis, drawing on tools and insights from earlier literature on market economies.

In the following analysis, I offer an alternative formulation of parallel markets in CPEs, which departs radically in conception from the Wellisz and Findlay model and, I may suggest, captures in a more natural fashion the essence of the problem at hand. This formulation is based on two important features central to the emergence of parallel markets in CPEs. First, the analysis incorporates the widely-held idea among Western experts that profit-seeking private firms can operate technically more efficiently than the
bureaucratically-run state enterprises. Second, the report takes explicit account of the fact that Soviet-type economies are subject to price controls and quantitative allocations which provide incentive for underground economic activity.

An important issue that any general-equilibrium analysis of CPEs must confront concerns the disposal of the income generated by capital. In these economies, capital (the so-called "means of production") as well as the income attributable to it belongs to the state. Therefore, we must specify how the state spends this income. In reality, a large part of the state's civil outlays in the Soviet Union goes into capital formation. For this reason, the most realistic approach will be to assume that the income attributable to capital is spent on capital accumulation.

Explicit recognition of the fact that the state uses its income for capital formation requires a dynamic formulation of the problem. As this task is complicated, the analysis below is developed in two stages. In Part I, I make the simplifying assumption that state revenues are spent on bureaucrats' salaries. This assumption enables me to analyze the implications of the parallel market in a static framework. In Part II, I introduce the more realistic assumption that the state uses its revenues for capital formation. The analysis in this part is based on a two-period model. Finally, in Part III, I construct a computable general equilibrium model of the Soviet economy. The report has been written in such a way that parts I and II can be read independently of each other. Part III requires some prior knowledge of Part II.
Part I

The Parallel Market: A Static Analysis

This part of the report is divided into 5 sections. Section 1 states and examines a CPE model, assuming that the parallel market does not exist. In spirit, this model resembles the Lange (1936)-Lerner (1934) idealization of the Soviet economy. At this stage, no price-quantity restrictions are introduced. Section 2 introduces the parallel market with illegal firms utilizing productive factors diverted from enterprises in the legal, planned economy but with enforcement expenditures directed at the illegal firms. The model in this section is driven by the fact that private firms can operate more efficiently than state enterprises. Section 3 extends the analysis to the realistic case where the state seeks to monitor the activities of legal enterprises aimed at seeking central allocations of the factors of production for profitable diversion to illegal firms. Section 4 studies the parallel market in a setting where the initial equilibrium is characterized by quantitative allocations at controlled prices and, consequently, the economy suffers from imbalances in demand and supply. Finally, Section 5 provides a summary of Part I.

1. An Initial Equilibrium

Consider a centrally planned economy endowed with two factors of production, labor (L) and capital (K). The economy produces two goods, 1 and 2, via production functions that are linear homogeneous in the two inputs. Good 1 is relatively labor intensive.

Assume that labor is supplied by private individuals while capital is owned by the state and that the income generated by each factor accrues to its
owner. As mentioned in the introduction, the state's income is used entirely to pay the salaries of bureaucrats. Let the utility functions of workers as well as bureaucrats be homothetic. Bureaucrats have a stronger preference for the capital intensive good than workers. That is to say, for a given marginal rate of substitution, bureaucrats' demand for good 2 relative to good 1 is higher than that of workers.

In order to analyze the implications of the parallel economy, we need to establish an initial equilibrium. I postpone the discussion of the implications of price controls and quantitative allocations until Section 4. For the present, I assume that the state's planning agency chooses various shadow prices so as to establish an equilibrium characterized by full Pareto efficiency. Thus, the shadow price of good 1, \( p_1 \) \((i = 1,2)\), the shadow rental on capital, \( r \), and the wage rate, \( w \), satisfy the relationship

\[
(1) \quad p_1 = c_i(w, r) \quad i = 1,2
\]

where \( c_i(w, r) \) represents the average and marginal cost of producing good \( i \).

Given linear homogeneous production functions, the \( c_i(w, r) \) will also be linear homogeneous in their arguments. Therefore, we can solve the marginal-cost-pricing conditions to obtain \( w \) and \( r \) as linear homogeneous functions of \( p_1 \) and \( p_2 \). Formally, we can write \( w = w(p_1, p_2) \) and \( r = r(p_1, p_2) \). These two equations allow us to represent the economy's GDP by \( r(\cdot)K + w(\cdot)L \). The first partial of this GDP function with respect the \( i \)th price yields the equilibrium output of the \( i \)th good. Letting \( X_i \) denote the output of good \( i \), we have

\[
(2) \quad X_i = r_i(p_1, p_2)K + w_i(p_1, p_2)L \quad i = 1, 2.
\]

where \( r_i(\cdot) \) and \( w_i(\cdot) \), respectively, denote the first partials of \( r(\cdot) \) and \( w(\cdot) \) with respect to the \( i \)th argument.

Equations (1) and (2) summarize the production side of our economy.
Next, let us consider the demand side. As noted earlier, bureaucrats as well as workers have homothetic preferences. Denoting the (government) bureaucrats' and workers' demands for good \(i\) by \(G_i\) and \(D_i\), respectively, and making use of the homotheticity assumption, we can write:

\[
G_i = r(p_1, p_2)K/g_i(p_1, p_2) \quad i = 1, 2
\]

\[
D_i = w(p_1, p_2)L/d_i(p_1, p_2) \quad i = 1, 2
\]

where \(g_i(.)\) and \(d_i(.)\) are linear homogeneous and their first partials, \(g_{ij}(p_1, p_2)\) and \(d_{ij}(p_1, p_2)\), are positive for \(i = j\) and negative for \(i \neq j\).

In intuitive terms, \(p_1/g_i(.)\) represents the proportion of bureaucrats' income spent on good \(i\). A similar interpretation applies to \(d_i(.)\). Remembering that bureaucrats have been assumed to have a stronger preference for good 2 than workers, we must have \(G_2/G_1 > D_2/D_1\) or, equivalently, \(g_2/g_1 > d_2/d_1\).

We can now close the model by invoking the usual market-clearing condition:

\[
X_1 = G_1 + D_1
\]

The system represented by (2)-(5) consists of 7 equations in 7 endogenous variables, namely, \(X_1\), \(X_2\), \(G_1\), \(G_2\), \(D_1\), \(D_2\) and \(p_1/p_2\) (= \(p\)). We can substitute for \(X_1\), \(G_1\), and \(D_1\) from (2)-(4) into (5) and solve the resulting equation for \(p_1/p_2\). Once we have \(p_1/p_2\), we can substitute it back into (2)-(4) to obtain the equilibrium values of the remaining variables. Throughout this part of the report, we will think of good 2 as the numeraire good. In our graphical analysis, we will explicitly set \(p_2 = 1\) and \(p_1 = p\), although in the algebraic analysis \(p_1\) and \(p_2\) will appear separately for ease of exposition.

The equilibrium just described will be called the "initial" equilibrium. In Figure 1, \(TT'\) shows the economy's transformation curve. \(U^G_1\) and \(U^G_2\) represent bureaucrats' indifference curves while \(U^D_1\) and \(U^D_2\) show workers' preferences. Note that if rental income is redistributed to workers, the
equilibrium will obtain at point C. Similarly, if all income is spent according to bureaucrats' preferences, equilibrium will be given by point B. In the present case, the equilibrium production point will be somewhere between C and B. Let this point be represented by \( \tilde{L} \). The equilibrium price ratio, \( \tilde{p} \), will equal the slope of the transformation curve at \( \tilde{L} \). Given this price, we can uniquely determine the ratios in which the two goods are demanded by bureaucrats and workers. These ratios are shown by rays OG and OD. Constructing the parallelogram \( O\tilde{G}\tilde{L}\tilde{D} \), we can obtain \( \tilde{G} \) and \( \tilde{D} \) as the consumption points of bureaucrats and workers, respectively. It is evident that the precise location of points \( \tilde{L} \), \( \tilde{G} \) and \( \tilde{D} \) will depend on the distribution of GDP between bureaucrats and workers. The larger the bureaucrats' share in income the closer will \( \tilde{L} \) and \( \tilde{G} \) be to B and farther will \( \tilde{D} \) be from C.

It is useful to conclude this section by summarizing the institutional arrangements in the initial equilibrium. We have postulated an economy in which capital is owned by the state. The rental on capital accrues to the state while wage income goes to workers. The state's income is used entirely for paying the salaries of bureaucrats. Bureaucrats have a stronger preference for capital intensive goods than workers. Based on its knowledge of tastes, technology and endowments, the central planning agency determines the appropriate shadow prices for goods and inputs and tells the state enterprises to follow the marginal-cost-pricing rule. Labor is paid according to the shadow wage rate determined by the planning agency while bureaucrats receive the residual. The output is then sold through the state's retail shops at the shadow prices just mentioned.\(^{12}\)

2. A Simple Parallel-Market Model

In this section, I introduce production activities of illegal firms into the economy just outlined. As noted earlier, a description of these
activities can be found in the studies by Simes (1975), Katsenelinboigen (1977), Grossman (1977) and others. Of particular interest in the present context is the following excerpt from Grossman.

Last, there are the underground entrepreneurs in the full sense of the term: that is, individuals who promote and organize production on a substantial scale, employ the labor of others, obtain materials and machinery on the black market, and distribute their output widely...The products involved are often consumer goods (garments, footwear, household articles, knickknacks, and the like) but can be producer goods as well.

In order to model illegal activities properly, it is important to introduce the government's enforcement policy explicitly. I shall consider two types of enforcement policies. In this section, enforcement will be directed against illegal firms while in Section 3, it will be directed against enterprises which seek the state's resources for diversion to the parallel market.

As noted in the introduction, I draw upon the popular notion among the Western Sovietologists that private firms which are motivated by profit seeking do not suffer from the kind of X-inefficiency that bureaucratically run state enterprises do. I capture this notion by assuming that there are potential private entrepreneurs who can produce good 1 more efficiently - in Hicks-neutral sense - than state enterprises. In formal terms, these entrepreneurs are assumed to face the unit-cost function

\[ c_1^P(w, r) = (1 - \alpha) c_1(w, r) \]

where "P" stands for "parallel" and \( \alpha \) is a constant between 0 and 1. Recall that \( c_1(.) \), introduced in equation (1), is the unit-cost function facing state enterprises in sector 1.
It is evident that given fixed endowments of labor and capital, illegal firms will have to rely on state enterprises for their input supplies. From the viewpoint of modeling, this fact raises the important question: What is the mechanism by which resources are diverted from state enterprises to the parallel market? In what follows, I attempt to answer this question in the simplest possible manner.

Consider first the planning agency's problem. Taking the enforcement policy as given, the planning agency chooses the shadow prices of goods and inputs so as to ensure full employment of resources and equality of demand for and supply of goods. It then allows workers to allocate themselves freely at the chosen shadow wage and provides enterprises with as much capital as they demand at the shadow rental rate.

Given that illegal firms are technically more efficient than state enterprises, they can offer a higher wage than the latter. Therefore, labor is attracted to the parallel economy. Similarly, illegal firms can offer a higher rental on capital than that charged by the state. This fact encourages enterprise managers to obtain more capital from the state than they need and divert it to the parallel market.

It is evident that if the state did not have an enforcement policy, illegal firms will wipe out state production entirely. Therefore, it is essential to introduce the state's enforcement policy explicitly. I assume that the state maintains an enforcement agency entrusted with the task of apprehending illegal firms. For simplicity, it is assumed that the enforcement agency has a fixed quantity of resources at its disposal. These resources are separate from the labor and capital used in goods production.

Let us denote the probability of detection of a firm by \( q \). A key assumption of our analysis is that \( q \) is a positive function of the
economy-wide ratio of illegal-to-total output of good 1, \( x \). Thus, we will write
\[
q = q(x) \quad 0 \leq x \leq 1; \quad q(0) = 0 \text{ and } q(1) = 1.
\]
\[
q'(x) > 0 \text{ for } 0 < x < 1.
\]
The assumption \( q'(x) > 0 \) is necessary to ensure coexistence of legal and illegal production and can be justified on the ground that the vigor of the state’s enforcement policy is a positive function of the ratio of illegal output to the total output.\(^{14}\) Alternatively, we could argue that as the proportion of illegal output expands, illegal firms become more visible to the enforcement agency. For simplicity, I assume that the penalty on a firm caught operating illegally is complete confiscation of its output.\(^{15}\)

We can now proceed to determine \( x \), the ratio of illegal-to-total output. Let the wage and rental rate paid by an illegal firm not apprehended by the enforcement agency be \( w^P \) and \( r^P \), respectively. Given our penalty assumption, the wage and rental paid by a firm which gets caught will, of course, be 0. Therefore, the expected wage and rental rate in the parallel market may be written \( (1-q)w^P \) and \( (1-q)r^P \), respectively. Assuming that workers and enterprise managers are risk neutral, we obtain \( (1-q)w^P = w \) and \( (1-q)r^P = r \). Perfect competition among firms not caught will lead to \( c^P_1(w^P, r^P) = p_1 \) or, equivalently, \( c^P_1(w, r) = (1-q)p_1 \). This last equation, combined with (1) and (6), yields
\[
(7) \quad q(x) = \alpha
\]
Intuitively, the cost advantage enjoyed by private firms is exactly offset by higher input prices facing them.

It is worth emphasizing that the equilibrium value of \( x \) turns out to be a function of \( \alpha \) alone. Interestingly, it does not depend even on the relative price ratio. The reason for this result is that neither the proportionate
cost advantage to illegal firms nor the proportionate penalty in terms of higher input prices depends on the price ratio.

In order to determine the equilibrium values of other variables, let us introduce the GDP function. The GDP in the present economy consists of factorial incomes plus the revenue raised by the state via penalties. As the expected factor prices in the illegal sector equal actual factor prices facing state enterprises, we can represent the GDP by \( r(.)K + w(.)L + q(\cdot)p_1 x X_1 \).

Note here that \( x X_1 \) is the output produced in the parallel economy. Differentiating the GDP function partially with respect to prices, we obtain total outputs.

\[
\begin{align*}
X_1 &= r_1(p_1, p_2)K + w_1(p_1, p_2)L + q(x)x X_1 \\
X_2 &= r_2(p_1, p_2)K + w_2(p_1, p_2)L
\end{align*}
\]

On the demand side, the workers' demand function continues to be given by equation (4). Assuming that the state treats the revenue generated by the enforcement agency in the same way as rental income, the bureaucrats' demand function, (3), must be replaced now by

\[
(3') G_i = [r(p_1, p_2)K + p_1 q(x)x X_1]g_1(p_1, p_2)i = 1, 2
\]

Equation (5) continues to give the market-clearing condition.

The system embedded in (7)-(9), (3'), (4) and (5) consists of 8 equations and can be solved for 8 endogenous variables, \( x, X_1, X_2, G_1, G_2, D_1, D_2 \) and \( p_1/p_2 \). We shall refer to the solution to this system as parallel equilibrium.

At this point, it is useful to comment briefly on the output equations (8) and (9). Comparing them with equations (2), it is evident that at a given price ratio, the allocation of labor and capital between goods 1 and 2 in the initial and parallel equilibria is exactly identical. The reason for this result is that for a given price ratio, the (expected) wage and rental rate which guide the allocation of resources between the two goods is the same in
the two equilibria. As long as state enterprises continue to produce good 1, the wage and rental rate facing state enterprises and hence the expected wage and rental rate in the parallel economy continue to be determined by equation (1). Of course, the higher technical efficiency of private firms implies a net gain in the output of good 1. More precisely, given the cost differential shown in equation (6), production of quantity \( x \cdot X_1 \) in the parallel market results in a sacrifice of only \((1 - \alpha)x \cdot X_1\) amount in the "First Economy". Therefore, the presence of private firms leads to a net addition of \( \alpha \cdot x \cdot X_1 \) \((=q(.)x \cdot X_1)\) amount to the total quantity of good 1.

Figure 2 compares the initial and parallel equilibria graphically. For notational convenience, I use tilde (\( \tilde{\cdot} \)) and hat (\( ^\sim \)) to distinguish the equilibrium values of various variables in the initial and parallel equilibrium, respectively. Point \( \tilde{L} \) represents the production point while \( \tilde{G} \) and \( \tilde{D} \) are consumption points in the initial equilibrium. Holding the price at \( \tilde{p} \), let us introduce the parallel market. Since \( q(0) = 0 \) and \( \tilde{p}_1 = c_1(\tilde{w}, \tilde{r}) \), the expected profit on the first unit of illegal production, \([1-q(0)]\tilde{p}_1 - (1 - \alpha)\cdot c_1(\tilde{w}, \tilde{r})\), is positive. Resources will be drawn into the parallel market and due to the higher technical efficiency of private firms, production will move out of \( \tilde{T}T' \). Holding the price ratio temporarily fixed at \( \tilde{p} \), the parallel market activity will push the economy's production point directly to the right of \( \tilde{L} \) to, say, \( R \). At point \( R \), workers' income is unchanged while that of bureaucrats increases by the amount of penalties, \( \tilde{L}R \). As the bureaucrats will spend only a part of their income from penalties on good 1, there will be an excess supply of that good at price \( \tilde{p} \). It follows that the price of good 1 will fall and the economy will move from \( R \) in the northwestern direction. In the parallel equilibrium, production will take place at a point such as \( \tilde{P} \). The new equilibrium price, \( \tilde{p} \), will be given by the slope of \( \tilde{T}T' \) at point \( \tilde{Q} \).
where \( \hat{Q} \) lies horizontally to the left of \( \hat{P} \). The state will collect \( \hat{P}Q \) in penalties. As \( \hat{p} \) is less than \( \hat{p} \), the consumption rays will flatten down to \( OG' \) for bureaucrats and \( OD' \) for workers. The new consumption points, \( \hat{G} \) and \( \hat{D} \), are obtained by completing the parallelogram \( \hat{OGPD} \).

The most striking result is that the presence of the parallel market is detrimental to the welfare of workers and beneficial to bureaucrats.\(^{16}\) The reason for this result is that the presence of the parallel market causes the price of good 1 to fall so that the real income of labor falls unambiguously via the Stolper-Samuelson effect. By contrast, bureaucrats’ income rises due to an increase in the rental price of capital as well as the revenues raised in terms of penalties.

This result leaves the question of the overall effect of the parallel market on social welfare unanswered. There are essentially three possibilities in this regard. First, we could assume that social welfare depends solely on the welfare of workers. In this case, the parallel market will be detrimental to social welfare. Second, we could assume that social welfare depends on bureaucrats’ utility. Under this assumption, social welfare will improve with the emergence of the parallel market. Finally and more realistically, social welfare may be assumed to depend on the welfare of both sets of consumers. In this case, the effect of the parallel market on social welfare will be ambiguous in general. However, it can be shown that appropriate tax-subsidy schemes can be designed to improve the welfare of each entity in the presence of the parallel market. On this basis, one will conclude that the parallel market is (at least potentially) welfare improving.

Our model can be explored further by considering the comparative statics effects of a change in the enforcement policy. Formally, this task is accomplished by shifting the \( q(.) \) function. Without presenting all the
details of this exercise, it is worthwhile to note that a stricter enforcement in the present model leads to an increase in the official GDP. This result is consistent with recent Soviet experience. For instance, in the late 1970’s Brezhnev’s policy to control the parallel market was ineffective. When Andropov came to power, he cracked down on this market and the officially reported output showed a significant jump. This experience was repeated when Gorbachev replaced the weak leadership of Chernenko.

Before concluding this section, it is worthwhile to outline two generalizations of our model. First, the model is easily modified to allow for parallel markets in both sectors. In this case, introduction of parallel markets at a given price ratio will lead to an expansion of both goods. In terms of Figure 2, point R will lie strictly northeast of $\tilde{L}$. Depending on whether this point happens to be above or below ray $OL$ (not drawn), the shadow price of good 1 will rise or fall. In the former case, the real wage will rise unambiguously. Thus, once we allow for parallel markets in both sectors, it is possible for the welfare of workers to rise. Furthermore, since the state gets to collect penalties, bureaucrats’ welfare could also improve despite a decline in the real value of rental income.

The second extension concerns inclusion of enforcement costs into the model. This task can be accomplished most simply by assuming that the probability of detection also depends on the number of inspectors (i.e., amount of labor) engaged in enforcement activity. In this setting, a Rybczynski effect will accompany the introduction of the parallel market. Thus in Figure 2, at the initial prices, the economy will end up at a point northeast of $R$. If enforcement employs a sufficiently large amount of labor, the net result may be a shortage of good 1 at the initial prices and the planning agency will have to raise the relative price of that good in order to
clear the market. Once again, we will have a reversal of the results derived from the simple model.

3. **Penalties on "Resource Seeking"**

In the last section, it was assumed that the state does not monitor the resource-seeking activities of enterprises. We now modify this assumption and introduce the monitoring of resource-seeking activities of enterprises. In order to keep matters simple, I drop the enforcement activity directed at illegal firms and concentrate exclusively on the monitoring of enterprises. It may be pointed out at the outset that my objective here is merely to indicate the manner in which the monitoring of enterprises can be incorporated into the analysis. Therefore, I will not pursue the implications of such monitoring in detail.

For simplicity, let us assume that resource-seeking activities take place in sector 1 only. It is a fairly straightforward matter to extend the analysis to allow for resource-seeking in both sectors. The state can monitor enterprises in at least two ways. First, based on the knowledge of production technology and enforcement policy, it can form an idea of the capital-output ratio in actual production. Comparing this ratio with the officially reported one, it can arrive at an estimate of the amount of capital diverted to the parallel market. Second, the state can conduct on-site inspections to detect discrepancies between the actual volume of capital and that on official records.

The outcome of the state's enforcement policy depends critically on whether it monitors the diversion of capital at the aggregate level or at the level of individual enterprises. In the former case, an individual enterprise has no control over the probability of detection whereas in the latter case, its actions do influence that probability. I consider briefly each type of
In the case when the state is able to monitor only the aggregate capital-output ratio, raids are conducted randomly. Therefore, we can assume that all enterprises face a uniform probability of detection. In order to ensure an interior solution, we must further assume that this probability is an increasing function of the overall ratio of diverted-to-total capital in sector 1. The assumption can be justified on the ground that the frequency of raids is an increasing function of diverted-to-total capital in sector 1. Upon detection, a penalty is assessed on the diverted capital at the ad valorem rate γ.

I will assume that as a part of their resource-seeking strategy, enterprise managers replace the planning agency’s directive to equate the marginal cost to price by the profit-maximization objective. Using superscripts F and P to distinguish the variables in the first and parallel economies, respectively, we can represent the jth enterprise’s expected-profit function by

\[ \rho_j = p_1 G_1(k_{1j}, l_{1j}^F) + (r^P - r)k_{1j}^P - rL_{1j}^F - wL_{1j}^F - \gamma r m(K_1^P/K_1^F)k_{1j}^P \]

where \( G_1(.) \) is the production function facing enterprises in sector 1, \( r^P \) is the rental rate in the parallel market, \( m(.) \) is the probability of getting caught diverting capital to the parallel market, and \( K_1 = K_1^F + K_1^P \). The first term in the profit function represents the revenue earned on the output produced. The second term is the profit from diverting capital to the parallel market. The third and fourth terms are costs of producing the official output. Finally, the last term represents the expected penalty.

Maximization of \( \rho_j \) with respect to \( L_{1j}^F, K_{1j}^F \) and \( k_{1j}^P \) respectively, yields

\[
\begin{align*}
(10a) \quad & p_1 \frac{\partial G_1(k_{1j}, l_{1j}^F)}{\partial l_{1j}^F} = w \\
(10b) \quad & p_1 \frac{\partial G_1(k_{1j}, l_{1j}^F)}{\partial k_{1j}^F} = r
\end{align*}
\]
\((10c) \quad r^P - r = \gamma r_m K_1^P / K_1\)

The first two of these conditions are standard. They imply that unit-cost pricing, shown in equation (1), continues to hold. According to condition (10c), competition among enterprises to divert capital leads to an equalization of the difference between the official and private rental rates and the penalty per-unit of diverted capital. The extra rental earned by capital in the parallel market flows back to the state in the form of penalties.

Conditions (10a)-(10c) hold for all firms. Therefore, we can drop subscript \(j\) and apply the resulting conditions at the industry level. These three conditions along with two similar conditions each for the parallel market and sector 2, two full employment conditions, and three production functions sum up to 12 equations which determine the supply side of the economy. Given factor endowments and commodity prices, they can be solved for the 12 endogenous variables which include three outputs \(X_1^F, X_1^P\) and \(X_2\), three labor allocations \(L_1^F, L_1^P\) and \(L_2\), three capital allocations \(K_1^F, K_1^P\) and \(K_2\), legal wage and rental rates \((w, r)\) and the parallel market rental rate \(r^P\). The model can be closed by adding the demand side in the usual fashion.

Some of the important implications of this model can be derived in a straightforward manner. First, since the rental on capital facing private firms is higher than that facing state enterprises, the rates of technical substitution in the two sectors will fail to equalize. Second, at a given price ratio, an increase in the average rental rate in sector 1 will lead to a higher labor-capital ratio in that sector than the one obtained in the initial equilibrium of Section 1. Finally, as long as the presence of the parallel market leads to a decline in the relative price of good 1, the income distribution results derived in the last section will continue to hold. The
reason for this last conclusion is that conditions \( p_1 = c_1(w, r) \) continue to hold in the present model so that the wage and rental rates remain tied to the goods prices in the usual Stolper-Samuelson manner.

Finally, let us consider briefly the case when the state monitors the capital-output ratios of individual enterprises so that the probability of detection depends on the enterprise-level ratio of diverted-to-total capital. Formally, in the profit function given by \( \rho_j \), \( m(K_1^P/K_1^j) \) must be replaced by \( m(K_1^P/K_1^j) \). Making this substitution and maximizing \( \rho_j \) with respect to \( L_{1j}^F \), \( K_{1j}^P \) and \( K_{1j}^P \), we have the following conditions.

\[
\begin{align*}
(11a) & \quad p_1 \frac{\partial G_1(K_{1j}^P, L_{1j}^F)}{\partial L_{1j}^F} = w \\
(11b) & \quad p_1 \frac{\partial G_1(K_{1j}^P, L_{1j}^F)}{\partial K_{1j}^P} + \gamma \cdot r \cdot m \cdot (K_{1j}^P/K_{1j}^j) \cdot (K_{1j}^P/K_{1j}^j)^2 = r \\
(11c) & \quad r^P - r = \gamma \cdot r \cdot m(\cdot) + \gamma \cdot r \cdot m'(K_{1j}^P/K_{1j}^j). (K_{1j}^P/K_{1j})^2
\end{align*}
\]

Condition (11a) is standard and needs no further comment. Condition (11b) has one extra term. This term captures the favorable effect of increasing \( K_{1j}^j \) on the probability of detection. According to (11c), the extra revenue earned by diverting one more unit of capital to the parallel market must equal the penalty on this unit \((=\gamma \cdot r \cdot m(\cdot))\) plus the increase in penalties \((=\gamma \cdot r \cdot m'(K_{1j}^P/K_{1j}^j))\) on the pre-existing units due to an increase in the probability of detection.

In the present model, enterprises are treated symmetrically. Therefore, (11a)-(11c) apply to all enterprises. If we drop subscript \( j \), the resulting equations will give us industry-level equilibrium conditions in sector 1 of the First Economy. Once again, these three conditions can be combined with the other equations of the model in the manner described for the previous case to obtain the parallel market equilibrium.

4. Price Controls and Quantitative Allocations

The initial equilibrium introduced in Section 1 provides the simplest
possible framework necessary to analyze the parallel market activities arising due to higher efficiency of private firms. A possible criticism of this equilibrium is that it ignores the existence of price controls and quantitative allocations that are so characteristic of Soviet-type economies. In the present section I modify the analysis to allow for these important features of CPEs. In particular, I present a parallel market model based on Grossman's (1983) observation on the Soviet economy that in the consumer goods sector "the official fixed individual retail prices tend to deviate from their actual equilibrium levels and, more often than not, tend to be too low."

Let us begin the analysis by outlining what we will call a "controlled" equilibrium. Assume for the moment that there are no illegal firms and that the state wishes to implement a plan which involves a smaller proportion of the labor intensive good than at \( \bar{L} \) in Figure 1. To keep matters simple, let this plan be represented by the point where bureaucrats' indifference curve is tangent to the production possibilities frontier. The results remain unchanged if the state chooses any other plan so long as the plan involves a smaller proportion of good 1 than at \( \bar{L} \).

Using a bar (\( \bar{\cdot} \)) to distinguish variable values at the controlled equilibrium, the production plan may be represented by point \( \bar{B} \) in Figure 3. Denote the shadow prices associated with this plan by \( \bar{p}_1/\bar{p}_2 \) (= \( \bar{p} \)), \( \bar{w} \) and \( \bar{r} \). At \( \bar{p} \), bundle \( \bar{B} \) will translate into \( O\bar{Y} \) of national income in terms of good 2. Letting \( O\bar{Y}_G \) be bureaucrats' income, draw line \( \bar{Y}_G V \) with a slope \( \bar{p} \). Intersection of this line with ray \( OG \) at \( \bar{C} \) will give the bureaucrats' consumption bundle. Subtracting the bureaucrats' income from total income and their consumption bundle from total output bundle, we can obtain \( O\bar{Y}_D \) and \( \bar{D} \), respectively, as the income and consumption bundle of workers. Since \( OD = \bar{B} \bar{G} \) (or equivalently \( OG = \bar{B} \bar{B} \)) and \( O\bar{Y}_D = \bar{Y}_G \bar{V} \) (or \( O\bar{Y}_G = \bar{Y}_D \bar{V} \)) by construction, the
line joining $\bar{Y}_D$ and $\bar{b}$ will have a slope equal to $\bar{p}$. Thus, if the state sells good 1 to workers at $\bar{p}$, bundle $\bar{D}$ will exactly exhaust private income.

Remembering that workers' tastes are biased in favor of good 1, however, the marginal rate of substitution at $\bar{D}$ will exceed $\bar{p}$. Put differently, there will be a shortage of good 1 at the official price $\bar{p}$. This shortage will pave the way for the parallel market to emerge.

In considering the parallel market in this section, I limit the analysis to the relatively simple enforcement policy employed in Section 2. Thus, the state is hypothesized to prosecute only the firms operating illegally. The resource diversion activity of enterprises is not subject to monitoring.

Initially assume that private firms do not differ from state enterprises in terms of production efficiency. At the initial equilibrium, the probability of detection is zero ($q(0) = 0$). Therefore, the wage and rental rate in the illegal sector at that point will be the same as that in the legal sector. The unit cost of production of the first illegal unit of good 1 will be $\bar{p}$. Remembering that at the initial equilibrium workers are willing to pay a price higher than $\bar{p}$, the potential expected profits from selling the first unit illegally are positive. The parallel market will emerge.

As illegal production expands in relation to legal production, the probability of detection will rise and the wage and rental rate paid by the successful firms in the parallel economy will have to rise to keep the expected input prices there equal to $\bar{w}$ and $\bar{r}$. Moreover, the relative price of good 1 in the parallel market, $p^P$, will decline as illegal production expands. In equilibrium, $p^P$ will equal the average cost of production in the parallel market. As in Section 2, we have $(1-q)w^P = \bar{w}$ and $(1-q)r^P = \bar{r}$ so that $p^P = c_1(w^P, r^P)$ which leads to

\[ p^P [1 - q(x)] = c_1(\bar{w}, \bar{r}) = \bar{p} \]
Introduction of the parallel market in the economy depicted in Figure 3 gives rise to two additional variables, \( x \) and \( P^p \). In order to determine the equilibrium values of these variables, we need two extra equations. The first of these equations is provided by (12) while the second one is obtained by setting the excess demand in the parallel market equal to zero.

In order to economize on space, I will not present the complete model algebraically. Instead, I proceed directly to intuitive and graphical analysis. The first step in this direction is to note that if the state wants the economy as a whole (including the parallel market) to produce at \( \bar{B} \) in Figure 3, the planning agency must continue to set its shadow prices at \( \bar{p}_r, \bar{w} \) and \( \bar{r} \). This observation follows from the fact that at \( \bar{p}, \bar{w} \) and \( \bar{r} \), the entire economy's GDP function in terms of planners' prices remains unchanged at \( w(p_1, p_2)L + r(p_1, p_2)K \). Intuitively, as in Section 2, for a given price ratio the introduction of the parallel market does not alter the overall allocation of resources between sectors 1 and 2.

This result allows us to deduce immediately that if the state continues to guide the economy's total production towards \( \bar{B} \), introduction of the parallel market will once again lead to a decline in the utility of workers. As in Sections 2 and 3, the parallel market redistributes income from workers to bureaucrats. The latter continue to receive \( rK \) in rental income and, in addition, collect \( q(.\)xX \) in penalties. From workers' viewpoint the penalties work as a tax in that they raise the effective price of good 1 facing them. For this reason and because the parallel market does not serve to increase the total availability of good 1, workers' utility declines in the new equilibrium.

The parallel market equilibrium is depicted in Figure 4 where I have also reproduced the various points of interest from Figure 3 without necessarily
reproducing the associated budget lines or indifference curves. As already explained, the economy's total-output bundle continues to be $\bar{B}$. Using a * to distinguish points associated with the parallel-market equilibrium, let us represent the output of good 1 in the parallel economy by $\bar{B}F^*$ where $F^*$ lies horizontally to the left of $\bar{B}$. Note that $F^*$ represents the output vector in the first economy. Suppose that penalties collected by the state from illegal firms amount to $F^*M^*$. Then $\bar{B}M^*$ will be the quantity of good 1 sold illegally and $M^*$ will represent the basket disbursed through the official distribution system.

Penalties $F^*M^*$ translate into $\bar{V}_G^*\bar{Y}_G^*$ amount of good 2. Note that this conversion is done at the parallel market price to be determined shortly. Recalling that the rental income has remained unchanged at $\bar{O}\bar{Y}_G$, bureaucrats' income in terms of good 2 will rise to $\bar{O}V^*_G$. As we will see shortly, workers' income valued at planners' prices will decline correspondingly.

Bureaucrats' consumption bundle can be obtained by drawing a line (not shown) through $\bar{Y}_G^*$ with slope $\bar{p}$ and locating that line's point of intersection with ray $\bar{O}G$. Thus, $G^*$ represents bureaucrats' consumption bundle. As expected, this bundle dominates bureaucrats' consumption bundle in the absence of the parallel market, $\bar{G}$.

Workers' consumption bundle is derived in two steps. First, subtract vector $\bar{O}G^* \cdot \bar{V}_G$ from vector $\bar{O}M^* \cdot \bar{Y}_G$. This task is accomplished by completing the parallelogram whose two sides are given by $\bar{O}G^*$ and $G^*M^*$ and obtaining point $C^*$. This point gives us the amount of good 1 obtained officially by workers at price $\bar{p}$. The second step is to add illegally purchased quantity $\bar{B}M^*$ of good 1 to that officially purchased and obtain $D^*$ as the consumption basket of workers. It is a straightforward matter to verify that $D^*$ will lie on ray $\bar{O}G$ and that this basket will be dominated by $\bar{D}$, the basket consumed in the
absence of the parallel market. We will also have $\bar{D}^* = \bar{G}^*$. At planners' prices, the value of basket $\bar{D}^*$ will be below that of basket $\bar{D}$ exactly by the amount of penalty, $\bar{Y}_D^{\bar{D}^*} (= \bar{Y}_D^{\bar{D}})$ where $\bar{Y}_D^{\bar{D}}$ is obtained by drawing a line (not shown) with slope $\bar{p}$ through $\bar{D}^*$.

Our remaining task is to determine the price of good 1 in the parallel market. For this purpose, note that the factorial income received by workers continues to be given by $O\bar{Y}_D$ in terms of good 2. Starting at $\bar{Y}_D$, workers can purchase bundle $B^*$ at the official price, where $B^*$ lies vertically above $C^*$ and on line $R\bar{Y}_D$. Note that $R\bar{Y}_D$ has a slope equal to $\bar{p}$ and, as in Figure 3, it passes through point $\bar{D}$. Beyond $B^*$, additional quantities of good 1 must be purchased at the higher illegal price. Since we have already determined $D^*$ as the final consumption basket, we can conclude that the relative price of good 1 in the parallel market will be given by the slope of line $B^*D^*$. Finally, the indifference curve of workers passing through $D^*$ will be tangent to line $B^*D^*$.

This analysis can be extended easily to incorporate the fact that private firms are more efficient than state enterprises. In this case, the production frontier in the presence of the parallel market shifts out horizontally. Consequently, the shadow price which maximizes bureaucrats' utility function will be below $\bar{p}$. The corresponding shadow wage and rental rates will be such that the real value of wage income will decline and that of rental income rise at planners' prices. Penalties on illegal firms which provide additional income for bureaucrats and raise the price of good 1 for workers will further benefit the former and hurt the latter.

Our model in this section is also modified easily to allow for the possibility that the state may choose the production plan so as to maximize bureaucrats' (or some other) utility function subject to the production
possibilities in the official economy. Starting with production plan $\tilde{B}$ in Figure 3, introduction of the parallel market shifts the official production point horizontally to the left. At this production point bureaucrats's marginal rate of substitution exceeds $\tilde{p}$ implying that the appropriate shadow price of good 1 is higher than $\tilde{p}$. A higher shadow price of good 1 also implies a higher shadow wage and lower shadow rental rate. Thus, in this case it is possible for the parallel market to result in a higher utility for workers provided penalties on illegal firms are not so large as to reverse the income distribution effect of a higher relative price of good 1.

5. Summary

In this part of the report, I have presented some static general-equilibrium models of the parallel market in the context of centrally planned economies. The models are based upon notions that private firms can operate more efficiently than bureaucratically-run state enterprises and that Soviet-type economies are often characterized by price controls and quantitative allocations. The analysis is highly stylized and is intended to capture only the most important features of the parallel market as described by Katsenelinboigen, Grossman, and others. A key feature of the analysis has been to link the resource-seeking activities of state enterprises to the operations of private firms in the parallel market. The models also take explicit account of the state's enforcement policy.

In the simplest model considered, the presence of the parallel market leads to a decline in the welfare of workers. The reason is that the additional goods supplied by more efficient private firms lower the relative price of labor intensive goods which, in turn, reduces the real wage. As workers derive their entire income by selling labor, their consumption possibilities necessarily decline. The overall impact of the parallel market
is, of course, favorable in the sense that it is possible to design tax-subsidy schemes that will allow both bureaucrats and workers to benefit from the parallel market.

I have also presented several modifications of the simple model. It is shown that if parallel markets exist in both sectors, it is possible for the income distribution result mentioned earlier to be reversed. Analogously, the presence of enforcement costs can give rise to the possibility that the parallel market raises workers' welfare.

The present part of the report has also developed a model in which the state chooses a plan which creates a shortage of the labor intensive good. This setup naturally paves the way for profitable illegal production. It is demonstrated that once again introduction of the parallel market proves harmful for workers. In the equilibrium that emerges, workers' wage income in terms of good 2 remains unaffected while they have to pay a higher price for illegally purchased quantities of good 1. The initial shortage of good 1 is cleared by a higher price (in the parallel market) rather than larger total supply.

According to our analysis, a more strict enforcement policy on the part of the state leads to an increase in the officially reported GDP. This conclusion is broadly consistent with reality as exemplified by significant jumps in the officially reported output after the weak governments of Brezhnev and Chernenko, respectively, were replaced by strong leadership of Andropov and Gorbachev.
Part II:
The Parallel Market: A Dynamic Analysis

This part of the report builds on the analysis in the previous part and develops the theory of parallel markets in a dynamic framework. The dynamic formulation is better able to capture some of the important institutional features of CPEs than the existing static models. Moreover, this formulation proves helpful in constructing a computable general equilibrium model of the Soviet economy in Part III.

I begin the analysis in Section 1 by outlining a simple two-period model with one consumption and one investment good. In Section 2, the model is modified to allow for the fact that in CPEs the state has a stronger preference for investment than private individuals. In Section 3, I introduce the parallel market and study its implications for the optimal plan and welfare. In Section 4, I consider the parallel market in a model with two consumption goods. Finally, in Section 5, I summarize the main conclusions of Part II.

1. A Two-Period Model

My intention in this section is to capture the state's role as the owner of capital and investor in the context of a second economy model in the simplest possible manner. For this reason, I limit the analysis to a two-period model which is tractable and lends itself to a diagrammatic exposition.

Let us begin by outlining the model in the absence of the second economy. Assume that there are two goods, a consumption good, $C$, and an investment good, $I$. Both goods are produced via linear homogeneous production functions.
There are two factors of production, labor and capital. Labor is supplied by private individuals and capital by the state. Individuals have identical homothetic preferences so that their combined demand can be represented by a well-behaved homothetic utility function.

\[ U = U(C_0, C_1) \]

where \( C_0 \) and \( C_1 \) represent the economy-wide consumption in periods 0 and 1, respectively. Frequently, I shall refer to periods 0 and 1 as current and future periods. For simplicity, it has been assumed that bequest does not yield any utility. None of our substantive conclusions are altered if we relax this assumption.

In this section, I shall assume that the state chooses its plan so as to maximize private utility. Thus, the planning agency's task is to maximize \( U(.\) subject to the following constraints.

\[ C_0 = F(L_{c0}, K_{c0}) \]
\[ I_0 = G(L_{i0}, K_{i0}) \]
\[ L = L_{c0} + L_{i0} \]
\[ K_0 = K_{c0} + K_{i0} \]
\[ C_1 = F(K_1, L) \]
\[ K_1 = K_0 + I_0 \]

Here \( I_0 \) stands for investment in period 0, \( L_{j0} \) and \( K_{j0} \) \((j = c, i)\) are labor and capital employed in sector \( j \) in period 0, \( L \) and \( K_0 \) are endowments of labor and capital in period 0, and \( K_1 \) is the amount of capital available in period 1. \( L \) and \( K_0 \) are exogenously given while \( K_1 \) is endogenously determined. It is assumed that the supply of labor in period 1 is the same as that in the initial period.

Equations (2) and (3) represent outputs of \( C \) and \( I \) in period 0. Both \( F(.) \) and \( G(.) \) are linear homogeneous in their arguments. It will be assumed
throughout that I is more capital intensive than C. Equations (4) and (5) are full employment constraints in period 0. Equation (6) represents the output of C in period 1 and equation (7) ensures full employment of capital in the same period. We can incorporate (7) into (6) but writing it separately allows us to derive the relationship between the shadow price of I_0 and the shadow rental rate on K_1 explicitly. Note that since capital is assumed to yield no utility by itself (i.e., the bequest motive is absent), no resources are devoted to its production in period 1.

The following are the first order conditions obtained by maximizing (1) with respect to C_0, I_0, L_0, K_0 (j = c, i), C_1 and K_1 subject to constraints (2)-(7).

(8) \( \partial U / \partial C_0 = p_0 \)
(9) \( p_1 = r_1 \)
(10) \( p_0 \partial F / \partial L_{co} = w_0 \)
(11) \( p_0 \partial F / \partial K_{co} = r_0 \)
(12) \( p_1 \partial G / \partial L_{io} = w_0 \)
(13) \( p_1 \partial G / \partial K_{io} = r_0 \)
(14) \( \partial U / \partial C_1 = p_1 \)
(15) \( p_1 \partial F / \partial K_1 = r_1 \)

Note that \( p_0, p_1, w_0, r_0, p_1 \) and \( r_1 \) are the shadow prices associated with constraints (2)-(7), respectively.

The system represented by (1)-(15) gives 15 equations in 15 variables: \( U, C_0, I_0, L_{co}, K_{co}, L_{io}, K_{io}, C_1, K_1, p_0, p_1, w_0, r_0, p_1 \) and \( r_1 \). The solution to this system can be supported by a set of competitive markets. More to the point, we can set \( p_0 = 1 \) and think of \( p_1, w_0, r_0, p_1 \) and \( r_1 \) as the relevant relative prices.

All of the first order conditions are intuitive except perhaps (9).
According to (9), the shadow price of the investment good in period 0 equals the shadow rental rate on capital in period 1. The reason for this equality is that by assumption the only utility investment yields is the utility of the output it generates in period 1. If we allow bequest to play a positive role in determining welfare, utility of investment will exceed the utility of output generated by it in period 1. In this case, the shadow price of the investment good will be higher than the shadow rental in period 1.

Alternatively, if we had a multi-period model, the price of the investment good would equal the net present value of the stream of returns generated by it over all the future periods.

Before we conclude this section, it is useful to compress the model into a smaller number of equations and present it graphically. For this purpose, let us solve constraints (4) and (5) and optimality conditions (10)-(13) for $C_0$ and $I_0$ as functions of $p_0$ and $p_1$. The solution may be written in terms of the production possibilities frontier and the standard tangency condition.

(16) $I_0 = f(C_0)f'(.) < 0, \ f''(.) < 0.$

(17) $f'(.) = -p_0/p_1$

Next, combine (6) and (7), (8) and (14), and (9) and (15), respectively, into

(18) $C_1 = F(K_0 + I_0, L)$

(19) $\frac{\partial U}{\partial C_0} = \frac{p_0}{p_1}$

(20) $p_0 \frac{\partial F}{\partial I_0} = p_1$

Equations (1) and (16)-(20) can be solved for 6 variables: $U, C_0, I_0, C_1, p_0/p_1,$ and $p_1/p_1$. Thus, our larger system of 15 equations is reduced to a system of 6 equations.

Figure 5 depicts the model as represented by equations (1) and (16)-(20) graphically. The production possibilities frontier in period 0 (equation
(16)) is represented by TT' in the first quadrant. In the second quadrant, we have a 45-degree line which helps us translate a given vertical distance into horizontal distance. In the third quadrant, we depict $C_1$ as a function of $I_0$ (equation (18)). At $I_0 = 0$, $C_1$ equals $C_0$. As we increase $I_0$, $C_1$ increases at a diminishing rate along curve FF'. Finally, in the fourth quadrant, NN'T' shows the tradeoff between $C_0$ and $C_1$ available to the economy. Indifference curves $U_1$ and $U_2$ depict preferences implied by (1).

The locus NN'T' in the fourth quadrant is derived as follows. Suppose the economy is at T' in period 0. At this point, investment is 0 so that $C_1$ will equal $C_0$. Letting OF = OT', we can obtain point N' as the combination of $C_0$ and $C_1$ available in the economy. Analogously, if the production point in period 0 is Q, consumption and investment in that period will equal $\tilde{C}_0$ and $\tilde{I}_0$, respectively. The investment will lead to a higher output in period 1 as indicated by point S in the third quadrant. We will obtain M as the feasible combination of $C_0$ and $C_1$. Proceeding in this manner, we can construct the curve NN'T' in its entirety. Note that this curve must be strictly concave to the origin.

The optimal consumption basket in the economy is given by the point of tangency between an indifference curve and the locus NN'T'. This point is labeled M in Figure 5. Point Q which lies directly above M gives optimal value of $I_0$. slopes along M and Q, respectively, give $p_1/p_0$ and $p_0/p_1$. Given optimal values of $C_0$, $I_0$ and $p_0/p_1$, we can go behind the transformation curve TT' to obtain optimal allocations of labor and capital and the various remaining shadow prices.

Institutional arrangements employed to reach the optimum may be described as follows. The planning agency gives output targets associated with point Q to enterprises in period 0. The state provides capital at the shadow price.
associated with constraint (5). Analogously, the wage rate is set equal to the shadow price associated with constraint (4). At these input prices, per-unit production costs of the investment and consumption goods equals the optimal values of $p_I$ and $p_0$, respectively. After workers are paid $w_0L$, the state receives the residual, $p_I I_0 + p_0 C_0 - w_0L (= r_0 K_0)$. I assume that the rental income in excess of investment costs ($= p_I I_0$) is redistributed among workers in a lump sum fashion. Thus, the total disposable income of workers, $w_0L + (r_0 K_0 - p_I I_0)$, is exactly equal to the value of consumption, $p_0 C_0$.

Income and expenditures are equalized for all agents in period 0. In period 1, there is no need for new investment. All resources are devoted to the production of the consumption good. The state redistributes all the income generated by capital among individuals which ensures equality of income and expenditures.

2. A CPE Model

Up to now we have assumed that the maximizes utility as represented by individual preferences. A more realistic assumption, however, is that the state is driven by a stronger desire for capital accumulation than private individuals. Accordingly, I shall now assume that the state maximizes a homothetic utility function which exhibits, for a given marginal rate of substitution, a higher $C_1/C_0$ ratio than the private utility function. This utility function will be written

\[ W = W(C_0, C_1) \]

In Figure 5, the optimal solution based on the state's preferences is given by point G. Note that $W_1$ is an indifference curve representing the state's preferences. The optimal plan in the current period is R. The state sets the rental rate on capital and the wage rate equal to the shadow prices.
associated with constraints (5) and (4), respectively. The relative price of current-period consumption is set equal to the slope of TT' at point R. After workers are paid their wages, the state receives the residual. All revenues in excess of the value of investment are redistributed among individuals in a lump sum fashion. The GDP, measured in units of $\frac{1}{s}$, equals $OY$ in Figure 1. Amount $OYG$ is used by the state for investment. The remainder, $YT'G$, equals wage bill plus transfer and translates into $YGR$ amount of $C_0$ at the shadow price associated with point R. As expected, demand and supply match.

At point R, the state engages in "forced savings". Left to themselves, individuals will save less and move to point Q. In terms of individual utility, forced savings are accompanied by a lower welfare than could be achieved otherwise.

The principal issue of interest now is whether the equilibrium just outlined is sufficient to encourage parallel-market production of the consumption good. The answer to this question is negative. Under the assumed institutional arrangements, individuals' disposable income in period 0 is just enough to purchase the available quantity of the consumption good in that period. There is no excess demand for $C_0$. Therefore, unless we introduce some additional elements into the model, forced savings by themselves will fail to give rise to the parallel market.

There are three alternative institutional arrangements which could give rise to the second economy. First, we could imagine that the state redistributes the entire rental income to individuals and then implements plan R in Figure 5 at price $YT'P$. In this case, individuals will want to purchase more $C_0$ than produced at R. The state will have to resort to quantitative allocations to force them to point R. There will be excess demand for the consumption good and the parallel market could emerge. Second, we could allow
for borrowing and lending. In this case, individuals will borrow against
future income so as to increase their current consumption. Once again excess
demand for $C_0$ will emerge. Third, we could hypothesize that the rental income
is insufficient to cover the desired investment and the state imposes a
turnover tax on consumption to bridge the gap. In this case, the official
price paid by individuals for $C_0$ will exceed the marginal cost of production
and illegal firms could flourish by evading the turnover tax. This last
alternative is the essence of the Wellisz and Findlay (1986) paper.

The first two of these alternatives are in conflict with the
institutional arrangements in CPEs. The state neither allows individuals to
make investment choices nor provides the facility to borrow for consumption.
The third alternative seems plausible but, at least in the Soviet case, the
assumption that rental income is insufficient to cover expenditures on
investment does not hold. The input-output table for the Soviet Union
constructed recently by Gallik et al for the year 1977 shows that the
depreciation fund and "profits" from state enterprises exceed the investment
demand. Typically, revenues from the turnover tax are needed to cover
expenditures on other items such as defense and public services.

In view of these considerations, we must look for further alternatives to
explain the existence of the parallel market in CPEs. We will pursue this
objective in the next two sections.

3. The Parallel Market when Private Production is More Efficient than
Bureaucratically Run State Enterprises

As in Part I, let us assume that private entrepreneurs are more efficient
than state enterprises. Assume further that private production is possible in
the consumption goods sector only. Formally, we can write,

\[ \beta^p(w_t, r_t) = (1-\alpha)\beta(w_t, r_t)t = 0, 1; 0 < \alpha < 1 \]
where function $\beta(.)$ represents the unit cost function associated with the production function shown in equation (2). The superscript $P$ distinguishes variables relating to the parallel economy.

Given the higher technical efficiency, private illegal firms can afford to pay higher wage and rental rates than those prevailing in the official economy. Therefore, labor as well as capital will be attracted to these firms. Resources will flow from the official to the parallel economy.

As in Part I, assume that the state maintains an enforcement agency entrusted with the task of apprehending illegal firms. Denote the probability of detection and conviction by $q$. Assume that upon conviction, a proportion of the output is confiscated by the authorities. In order to ensure coexistence of illegal firms and state enterprises, it must be assumed that either the probability of detection or the proportionate penalty or both rise with some measure of industry level illegal activity. For convenience, I assume that it is $q$ that varies with the level of illegal activity and set the penalty proportion equal to 1 (i.e., complete confiscation). The results remain unchanged if $q$ is assumed constant and the proportionate level of penalty is assumed to vary with illegal activity. Let us write

$$q_t = q(x_t, e_t)q_{x_t}q_{e_t}, \quad q_{x_t} > 0; \quad t = 0, 1$$

where $x_t$ is the ratio of illegal-to-total output of the consumption good and $e_t$, specified exogenously, is an index of the vigor of the enforcement policy in period $t$. Individual firms take $x_t$ and hence the probability of detection as given. Note that we are assuming that probabilities of detection in the two periods are independent of each other. This assumption simplifies the analysis considerably and enables us to represent the resulting equilibrium diagrammatically.

In order to determine $x_t$, let us denote the wage and rental rate paid by
an illegal firm not caught by the enforcement agency by $w_t^P$ and $r_t^P$, respectively. Given our penalty assumption, firms which get caught will fail to pay anything for labor and capital they employ. Therefore, the expected wage and rental rate in the parallel market will be $(1-q_t)w_t^P$ and $(1-q_t)r_t^P$, respectively. Assuming workers and enterprise managers are risk neutral, we obtain $(1-q_t)w_t^P = w_t$ and $(1-q_t)r_t^P = r_t$. Perfect competition among firms not caught will lead to $\beta^P(w_t^P, r_t^P) = p_t$ or, equivalently, $\beta^P(w_t, r_t) = (1-q_t)p_t$. This last relationship, combined with equation (22) and the fact that legal firms equate $\beta(w_t, r_t)$ to $p_t$, yields

$$q(x_t, e_t) = a$$

Intuitively, the cost advantage enjoyed by private firms is exactly offset by higher input prices facing them.

Equation (24) determines $x_t$ solely as a function of $a$. Interestingly, the solution value of $x_t$ does not depend on any of the prices. The reason for this result is that neither the proportionate cost advantage nor the proportionate penalty in terms of higher input prices depends on the level of prices. It deserves noting that as long as the level and technology of enforcement remain unchanged, $x_t$ will attain the same value in periods 0 and 1.

In order to consider the implications of the parallel market for the economy, I shall first assume that the state takes illegal production into account while choosing the optimal plan. The extension to the more realistic case when the state optimizes only over the legal sector will be presented later.

Consider Figure 6 which uses a construction similar to that in Figure 5. The current-period production plan in the absence of the parallel market is represented by $R$. This plan maximizes utility as determined by the state's
preference function. Suppose that we introduce the parallel market holding all shadow prices at their initial level. At \( x_0 = 0 \), the probability of detection is 0 which implies \( w^P = w \) and \( r^P = r \). Remembering that illegal firms are more efficient, their production costs at these prices will be lower than those of enterprises. Therefore, illegal production will emerge. As \( x_0 \) rises, however, the cost advantage will decline and ultimately disappear. In equilibrium, the cost advantage will be exactly offset by the proportionately higher input prices. The new production point of the economy, \( \tilde{R} \), will lie horizontally to the right of the initial point, \( R \).

As in Part I, at constant prices, introduction of the parallel market does not lead to any reallocation of resources between investment and consumption goods sectors. In equilibrium, \( q(\cdot) = \alpha \), \( w^P(1-q) = w \), and \( r^P(1-q) = r \). Using these relationships, equation (20) may be rewritten as

\[
\beta^P(w^P_t, r^P_t) = (1-\alpha)\beta(w^P_t, r^P_t) = \beta(w^P_t, r^P_t)
\]

That is to say, in equilibrium, illegal firms face the same cost function as enterprises. Hence as long as the shadow prices of goods and inputs are kept at the original level, emergence of the parallel market will not change the resource allocation between the two sectors. Of course, since private firms are more productive, the total output of \( C_0 \) will rise. This is the reason why \( \tilde{R} \) lies horizontally to the right of \( R \).

Proceeding in the manner just described, we can trace the entire frontier of the economy in the presence of the parallel market in period 0. This frontier is represented by \( TT' \) in Figure 6. It is evident from the discussion in the previous paragraph that the slope of a point on \( TT' \) will be the same as the slope of the point lying horizontally to the left on \( TT' \).

Next, let us consider the impact of the parallel market in period 1.
Recall that there is no production of the investment good in this period. Assuming for the moment that the level and technology of enforcement do not change, the parallel market will shift the FF' curves proportionately up as shown by FF'. Given this curve and TT, we can derive the economy's trade off between current and future consumption in the same way as in Figure 5. The new trade off is shown by curve NN' in the fourth quadrant of Figure 6.

I demonstrate in the appendix that the curve NN' is necessarily parallel to NN'. Therefore, if the state's preferences are homothetic, the new optimal plan chosen by the state will leave the current-to-future consumption ratio and the level of investment completely unaltered. The absolute level of consumption will rise in each period so that welfare will increase unambiguously.

In the event that the state's preferences are nonhomothetic, the current-to-future consumption ratio and the level of investment will change with the introduction of the parallel market. In particular, if these preferences tilt in favor of current consumption as income rises, the plan chosen in the presence of the parallel market will be characterized by higher current-to-future consumption ratio and lower investment than in the original equilibrium. In this case, welfare, as measured by private utility, will rise more than in the homothetic case.

Introduction of depreciation of capital stock at a constant exogenous rate leaves the conclusions in the previous two paragraphs unchanged. Essentially, depreciation causes both NN' and NN' to shift towards the origin in parallel fashion. Therefore, the ratio of current and future consumption changes only if the state's preference function is nonhomothetic.

Next, let us assume that the state chooses its plan by optimizing over the official economy alone. Since the parallel production withdraws resources...
from the official economy, the former's emergence will shift the official production frontier in period 0 inside $TT'$ as shown by $\bar{TT'}$ in Figure 7. Similarly, the production function in period 1 (not shown) will shift towards the horizontal axis. The combined effect of these shifts will be to move $NN'$ in parallel towards the origin to $\bar{NN'}$ (see the appendix for a proof). If the state's preferences are homothetic, there will be no change in the level of investment and, indeed, the economy will achieve the same equilibrium as when the state takes the parallel production fully into account.

In the event that the state's preferences are nonhomothetic and tilt towards present consumption with rising incomes, the inward shift in $NN'$ will lead to a rise in investment and a decline in the ratio of the present-to-future consumption level. Thus, the economy will move farther away from point $\bar{H}$, which maximizes private utility in the presence of the parallel market. If nonhomotheticity in the state's preferences is sufficiently strong, such a change can even lead to a decline in private utility relative to that achieved at $G$ in the absence of the parallel market.

In concluding this section, it may be noted that the results derived above depend critically on the fact that $x_t$, the ratio of illegal-to-total output, is the same in periods 0 and 1. Any changes in the value of $x_t$ across periods 0 and 1 will alter the ratio of present-to-future consumption even when the state's preferences are homothetic. Three possibilities may be mentioned in this regard. First, if enforcement becomes tighter in period 1, $x$ will be lower in that period than in period 0. The locus $\bar{NN'}$ will be flatter at each level of $C_0$ than that shown in Figure 6. The result will be a higher optimal ratio of current-to-future consumption and a lower investment. Second, if the probability of detection depends not merely on the ratio of
illegal-to-legal output but the absolute level of illegal activity as well, $x_t$ will have to be determined simultaneously with other variables rather than via equation (24) alone. In such circumstances, it is unlikely that $x_t$ will assume the same value in period 1 as in period 0. Third, if enforcement is directed against enterprises but not against illegal firms and workers, the price of capital relative to wages will be higher for firms than for enterprises. As a result, introduction of the parallel market at the initial shadow prices will alter the allocation of labor and capital between the consumption and investment sectors in period 0. Moreover, the value of $x_t$ will depend not merely on $\alpha$ but on the other variables of the model as well. In general, $x_t$ will be different in periods 0 and 1 and the emergence of the parallel market will alter the optimal ratio of outputs in periods 0 and 1.

4. The Parallel Market with Two Consumption Goods

It was shown earlier that in the absence of borrowing and lending a discrepancy between the present-to-future-consumption ratio chosen by the state and that desired by private individuals will fail to generate excess demands. Therefore, shortages which are often said to exist in the Soviet economy must be the result of imbalances within the same period across different sectors. In the present section, I focus on the analysis of illegal economic activities which emerge due to imbalances of this type.

In order to study the implications of sectoral imbalances, we must introduce at least two consumption goods. This modification complicates the two-period analysis considerably. Even in the absence of the parallel market, we have 18 endogenous variables. For this reason, I adopt an approach which allows us to determine current consumption and investment without having to determine future consumption. Although this approach is not very satisfactory
on theoretical grounds, I adopt it for two reasons. First, we have already highlighted the issues that arise within an explicit two-period model. Our concern in the present section being with sectoral imbalances within the same period, it is helpful to omit complications which arise from across-period considerations. Second, the approach adopted here lends itself directly to the construction of a computable general equilibrium (CGE) model of the Soviet economy in Part III.

We begin by assuming that there are two consumption goods, $C_1$ and $C_2$, and one investment good, $I$. Henceforth, all subscripts will refer to sectors; there are no time subscripts. Good 1 is to be identified with items such as food, textiles and apparel and construction which are typically in short supply in the Soviet Union and are subject to illegal production.

In the remainder of this part, the state's utility function is assumed to take the following (separable) form.

$$V = v(C_1, C_2)^{h} I^{1-h}$$

Essentially, we replace the current- and future-period consumption quantities in (21) by $v(.)$ and $I$, respectively. The value of parameter $1-h$ will depend on the importance assigned to future consumption and bequest. Thus, to some extent, the specification in (25) is a shortcut to capturing the relationship between present and future consumption highlighted in the previous sections. The main limitation of the approach is that it is unable to take into account the effects of changes in the future period on the current-period solution.24

In the absence of the parallel market, the planning agency's problem is to maximize $V(.)$ subject to the following constraints.

$$X_i = F(K_i, L_i) \quad i = 1, 2.$$  

$$I = I(I_1, I_2)$$  

$$X_1 = C_1 + I_1, i = 1, 2.$$
(29) \( K = K_1 + K_2 \)

(30) \( L = L_1 + L_2 \)

These constraints are similar to those introduced in Section 1 except that all variables refer to current period and that investment has been assumed to use goods 1 and 2 as inputs. The reason for this change in specification is that in the data used to construct the CGE for the Soviet Union, investment appears in the form of goods. None of the results change if we specify the production function for investment in terms of primary inputs. It may be noted that \( I(.) \) is linear homogeneous in its arguments. The choice variables in the problem are \( X_1, C_1, K_1, L_1, I_1 \), and \( I \).

Let us assume that the private utility function is also separable between current consumption and investment and is represented by

(31) \( Z = z(C_1, C_2)^m I^{1-m} \)

It is assumed that \( v(.) \) in (25) and \( z(.) \) in (31) are homothetic with the latter exhibiting a higher \( C_1/C_2 \) ratio than the former for a given marginal rate of substitution between the two goods. Thus, individuals have a stronger preference for good 1 than the state. (Recall that good 1 represents items that are typically in short supply in the Soviet Union and subject to parallel production.) We also assume that \( m > h \) reflecting the fact that individuals give more importance to current consumption than does the state.

In the absence of the parallel market, the state chooses the plan as described earlier. Individuals are paid a wage at the shadow price associated with labor constraint plus a transfer payment. The transfer payment consists of income generated by capital minus the expenditure on investment. Consumption goods are allocated at the shadow prices associated with equations (26) (or, equivalently, those associated with equations (28)).

Given that individuals have a stronger preference for good 1 than the
state, there will be a shortage of good 1 at the shadow prices charged by the latter. This fact is shown in Figure 8. TT' represents the economy's tradeoff between goods 1 and 2 after subtracting resources necessary to produce the optimal level of investment, I. Analogously, V₁ and Z₁ represent the state's and individuals' indifference curves given I = I*. Point Q represents the plan chosen by the planning agency. In conformity with our assumption regarding relative marginal rates of substitution of the state and individuals, indifference curve Z₁ is steeper than V₁ at point Q. Thus, there is excess demand for good 1 at price pp'.

This excess demand is sufficient to pave the way for the emergence of the parallel market. Before we move on to a discussion of the parallel market, however, let us introduce a modification of the model which allows us to incorporate bribes explicitly into the analysis. Henceforth, we will assume that there is a fixed number of bureaucrats in the economy entrusted with the task of formulating and implementing the plan. These bureaucrats include enforcement inspectors and have the same homothetic tastes as private individuals. Bureaucrats' salaries are paid out of the income generated by capital.

We can proceed now by assuming that there are potential illegal firms. To keep matters simple, let these firms face the same technology as enterprises. Also, let enforcement be directed against illegal firms. In the simulations presented in Part III, we will make the more realistic assumption that enforcement is directed against enterprises which divert capital to the parallel market. Denote the probability of detection and conviction of firm j by n_j and write

\[(32) \quad n_j = n(x, \mu_j) \quad n_x > 0, \quad n_{\mu_j} < 0\]

where x is the economy wide ratio of illegal-to-total quantity of good 1 sold
and \( \mu_j \) is per-unit bribe on illegal sales paid by firm \( j \). Firms can lower their probability of conviction by increasing the bribe paid to appropriate authorities. We will measure \( \mu_j \) in units of good 1. Note that individual firms take \( x \) as given but treat \( \mu_j \) as a choice variable.

Denote the wage, the rental rate on capital and the price of good 1 in the parallel market by \( w^P, r^P \) and \( p^P_1 \), respectively. Firms which are caught and convicted will fail to make payments. Therefore, assuming risk neutrality on the part of all agents, we must have \((1-n)w^P = w\) and \((1-n)r^P = r\). The profit function of a typical firm which is successful in evading detection may be written

\[
\pi_j = p^P_1 C^P_{1j} - \beta\left(\frac{w}{1-n_j}, \frac{r}{1-n_j}\right) C^P_{1j} - \mu_j p^P_1 C^P_{1j}
\]

where subscript \( j \) denotes the firm. Profits are maximized with respect to \( C^P_{1j} \) and \( \mu_j \). Assuming that all firms face the same input and output prices, they will choose the same \( \mu \). Therefore, we can drop subscript \( j \) and write the first order conditions as follows.

\[
(33a) \quad (1-\mu)p^P_1 = \beta(., \mu)/(1-n)
\]

\[
(33b) \quad p^P_1 C^P_{1} = -\beta(., \mu) n C^P_{1} / (1-n)^2
\]

\[
= -(1-\mu)p^P_1 C^P_{1j} \mu / (1-n)
\]

According to (33a), price net of the bribe is equated to the production cost per unit. In equation (33b), the second equality is derived by using (33a). The left-hand side in this equation represents the cost of raising \( \mu \) by one unit while the right-hand side gives the additional revenue (due to a decline in the probability of being caught) per-unit change in \( \mu \). Thus, (33b) says that the marginal cost of bribes must equal the marginal revenue from such a change. Note that using (33a), we can rewrite (33b) as

\[
(33b') \quad (1-n) = -(1-\mu) n (x, \mu)
\]

Equations (33a) and (33b') can be solved for \( x \) and \( \mu \) as functions \( p^P_1 \).
As in the previous section, it is easiest to consider first the case when the state takes the parallel market into account in choosing its plan. We already know that at a given set of prices, the parallel market will not alter the allocation of resources among the two consumption goods and the investment good. Therefore, the state's optimal plan and shadow prices will remain unchanged. In terms of Figure 8, TT' will continue to be the relevant transformation curve between goods 1 and 2. The state will choose to implement plan Q at price ratio pp' and the excess demand for good 1 will induce illegal firms to emerge.

Figure 9 depicts the equilibrium with parallel production. Point R is the production bundle in the official economy and RQ is the output of good 1 in the second economy. Income available for current consumption at the planners' prices is OY and equals payments made to labor and capital employed in producing basket Q. YY' is the penalty plus bribes paid by illegal firms. Starting at Y', individuals and bureaucrats can move to basket S at the official price. Beyond S, they must pay the higher parallel-market price indicated by line SS'. The parallel-market price itself is determined so as to ensure that the excess demand for good 1 is eliminated. Moreover, this price must equal the marginal rate of substitution in private consumption at point Q.

In the more realistic case when the state chooses its plan on the basis of the official economy alone, the analysis is altered in two ways. First, the official income is less than the total income. Therefore, the parallel market leads to a reduction in investment. (Recall that based on the utility function we have assumed, the state spends a constant proportion of the income on investment.) Second, for a given level of C₂ in Figure 9, the rate of product transformation in the official economy will be higher than that along
TT'. Therefore, the state's plan for current consumption will exhibit a higher ratio of good 1 to good 2 production. Both of these changes will be beneficial for private individuals.

Finally, suppose the state's enforcement policy is directed against enterprises rather than private firms. In this case, illegal firms will face a higher rental-to-wage ratio than enterprises. Therefore, the former will use more labor intensive technology than the latter and the allocation of resources between goods 1 and 2 will be altered. As this case is rather difficult to trace analytically, we will consider it in our simulation of the Soviet economy in Part III.

5. Summary

This part of the report has studied the implications of the parallel market for welfare and resource allocation in dynamic models. A major advantage of dynamic models is that they enable us to treat the disposal of the state's revenues (generated by capital) in a realistic manner. In CPEs, a large part of the revenue generated by capital is used for investment. Inclusion of this feature in the formal analysis necessarily requires a dynamic formulation of the problem.

The analysis in Sections 1 to 3 is based on a two-good, two-period model. The two goods are a consumption good and an investment good. It is shown in Section 2 that contrary to the existing impression in the literature (e.g., Wellisz and Findlay), "forced savings" by themselves are not sufficient to give rise to the parallel market in CPEs. Given that revenues generated by capital provide enough resources to the state for capital accumulation, and that individuals are not allowed to borrow for consumption, there will be no excess demand to induce illegal production activity.

The parallel market will emerge, however, if potential illegal firms are
more efficient than state enterprises. Assuming that preferences are homothetic and that the state chooses its plan to maximize social welfare based on officially produced goods, this type of illegal activity necessarily lowers investment and raises total consumption in the current period. In the case when the state takes the official as well as parallel production into account, the level of investment remains unchanged. In both cases, welfare rises in terms of private preferences.

The parallel market can also emerge if we have 2 or more consumption goods and the state fixes their relative prices at levels inconsistent with demand-supply balance. In this case, illegal firms profit from selling the good in excess demand at a price higher than that officially fixed. Section 4 has developed a model along these lines. The model allows for endogenous determination of bribes. It is shown that if the state optimizes over the first economy alone, emergence of the parallel market lowers investment and raises current consumption of the good in short supply. Welfare, as determined by private preferences, rises.
Part III

A Computable General Equilibrium Model of the Soviet Economy

In this part of the report, I present the results of a computable general equilibrium (CGE) model of the Soviet economy. The model employs an aggregated version of the recently constructed input-output table by Gallik et al (1984) for the year 1977. It must be emphasized at the outset that in constructing a CGE a series of restrictive assumptions have to be made. Therefore, the numbers reported below should be considered only as suggestive. 25

I first describe the basic steps in constructing the CGE and then summarize the results of simulations.

1 The Input-Output Table

As mentioned above, the 1977 input-output table constructed by Gallik et al formed the basis of the CGE model. The original table, at producer prices, has 16 sectors. For present purposes, these sectors were aggregated into 2 and labeled sectors 1 and 2. Sector 1 consists of textiles and apparel, food, trade and distribution, and construction while sector 2 contains all the other industries. According to the available evidence, industries included in sector 1 are often the subject of parallel market activity. Table 1 shows the two-sector aggregation after minor simplifications with respect to taxes and subsidies.

Table 1 was modified to take account of illegal economic activity by assuming that 10 percent of the total consumption in sector 1 originates in the parallel market. Incomes generated by illegal production activity were added to labor incomes. 26 The final outcome was Table 2 which formed the basis
of our benchmark equilibrium. These tables, the original 16 x 16 table and
some related details are available in diskette form from the National Council
for Soviet and East European Research.

2 The Model

The model is the same as described in Section 4 of Part II. The
following remarks cover some of the areas where confusion may arise.

1. The model has been modified to take intermediate inputs into account.
   As illegal production in sector 2 is ruled out by assumption, we must allow
   for a diversion not merely of capital but intermediate inputs from sector 2 as
   well.

2. It has been assumed that enforcement is directed against enterprises
   which divert capital and/or intermediate inputs but not against workers or
   illegal entrepreneurs. Thus, illegal firms face the same wage as legal
   enterprises but have to pay a higher price of capital. The result is that the
   former employ a more labor intensive technology than the latter. Bribes are
   explicitly incorporated into the probability-of-detection function.

3. The state is assumed to optimize over the official economy only.
   Thus, the plan is chosen to maximize the state's utility in terms of
   officially produced output.

3 Functional Forms

All the utility functions are assumed to be Cobb-Douglas. Production
functions for goods 1 and 2 are of the form

\[ X_j = \min \{ V_{A_j}(K_j, L_j) / A_{v_j}, M_{1j} / A_{1j}, M_{2j} / A_{2j} \} \quad j = 1, 2 \]

where \( M_{ij} \) (\( i, j = 1, 2 \)) is the quantity of good \( i \) used in \( j \) and \( A_{ij} \) is the fixed
input-output coefficient. \( V_{A_j}(.) \) represents value added in sector \( j \) and \( A_{v_j} \)
is the fixed coefficient between gross output, \( X_j \), and value added. It is
assumed that \( F_j(.) \) has the CES form. Thus, substitution is allowed between
labor and capital but not between value added and intermediate inputs.

The probability-of-detection function has the form

\[ q = Bx^A(1+p)^{-E} \]

where \( A, B \) and \( E \) are positive constants. \( E \) is larger than unity. An increase in \( A \) implies weaker enforcement. Variable \( p \) represents the proportionate bribe on the profit earned on the diverted capital or on the input of good 2. For simplicity, I have assumed the same probability function for diversion of capital and good 2. Using different probability functions for the two types of diversion will not make a significant difference to the results.

Investment, \( I \), is produced via a Cobb-Douglas production function using goods 1 and 2. Capital stock is assumed to depreciate at the rate of 10 percent per annum.

4. The Benchmark Equilibrium

We chose units of various inputs and outputs in such a way that their prices in the first economy equal unity in the benchmark equilibrium. We then chose values of various parameters so that the model reproduces the equilibrium as shown in Table 2. \( A, B, E \) and the parallel market price were calibrated by assuming that \( q \) and \( p \) are 0.10 and 0.20, respectively, in the benchmark equilibrium. There are no data available on these latter variables.

The values assumed seem plausible. Moreover, the basic message of the model does not change if we choose different values.

5. Results

Results of various simulations appear in Tables 3-5. In the interest of clarity and economy of space, I have presented the results with respect to only the most important variables. Note that due to the requirements of the computer program, the notation had to be changed slightly. Thus, I am now denoting net investment by \( IN \), consumption in sector 1 by \( C1 \), wage rate by \( W \),
rental rate by R, price of good 1 by P1, proportion of good 1 sold illegally by X, the parallel market price of good 1 by PP1, the probability of detection by QX and private utility by U. Parameter LBD, set exogenously, allows us to distinguish between production functions in the first and second economies for good 1.

Case 0 reports the levels of these variables in the benchmark equilibrium. All prices in the first economy are set equal to 1 by the choice of units. Net investment (IN) is calculated on the assumption that capital depreciates at the rate of 10 percent per annum. We set LBD equal to 1 initially implying that technology is the same in the first and second economies in the benchmark equilibrium. The price of good 1 in the second economy is approximately 11 percent higher than that in the first economy.

5.1 Gains from the Parallel Market

Our first simulation asked the following question: If there were no parallel market and we wanted private individuals to enjoy the same utility as in the benchmark equilibrium, how much extra capital would we need? The conclusion was that we would have to increase capital stock by approximately 2 percent. Thus, the presence of the parallel market is equivalent to a 2 percent increase in capital stock.

5.2 Changes in Enforcement and Bribe Parameters

Cases 1–6 in Table 3 report the effects of changes in parameters related to the probability-of-detection function. An increase in A (cases 1 and 2) and a decrease in B (cases 3 and 4) imply weaker enforcement. The first thing to note is that weaker enforcement has the largest proportionate effect on the extent of illegal activity. The response of all prices including the parallel market price is weak. The reason for this result is that sectors 1 and 2 do not differ a great deal in terms of capital-labor ratios. Therefore, the
production frontier is relatively flat and output changes do not result in a large price changes. Interestingly, the effect on P1 is positive for smaller changes in A and B (case 1 and 3) but negative for larger changes in the parameters. (cases 2 and 4). Expansion of the parallel market is also accompanied by a decline in investment and, therefore, an increase in present consumption. Utility, as measured by private preferences, rises.

Cases 5 and 6 in Table 3 look at the effects of changes in E. A rise in E implies that a given proportionate bribe becomes more effective in evasion. Not surprisingly, the results in these cases are similar to those in cases 1-4. The only difference is that P1 continues to rise as E is increased.

5.3 An Increase in the Efficiency of Illegal Firms

Cases 7-8 in Table 3 consider the effect of an increase in the efficiency of illegal firms. A 5 percent increase in efficiency leads to a 10 percent expansion of the parallel market. In other words, X rises from 0.10 to 0.11. The parallel market price declines, but only slightly. Case 8 looks at a more dramatic case where private firms become twice as efficient as state enterprises. Not surprisingly, the effects are dramatic as well. The parallel market more than doubles in size.

5.4 Evolution of the Economy with Enforcement Constant

Table 4 shows the evolution of the economy for 4 years following 1977. Case 0 is the benchmark equilibrium in terms of the levels of different variables and cases 1-4 correspond to years 1978-81 as predicted by the model. In these cases, we increase capital stock each period by the net investment in the preceding year and labor by an exogenous factor. According to the data, capital grows considerably faster than labor. This fact is reflected in a declining shadow price of capital and rising shadow wage rate. The relative price of good 1 which is labor intensive rises in the legal as well as illegal
economy. Illegal sector expands but, only slowly.

5.5 Evolution of the Economy when Enforcement becomes Weak Over Time

Finally, Consider Table 5. Here we allow the parameters in the probability-of-detection function to change as the economy evolves. Cases 1-4 allow enforcement to get progressively weaker. The result is a substantial expansion of the illegal sector. In the scenario considered, illegal sales of good 1 increase from 10 percent to 14 percent of total sales in 4 years. The price in the parallel market declines, but not very much.

Cases 5-8 look at the evolution of the economy when the probability of detection for a given proportionate bribe declines over time. The results are generally similar to those in cases 1-4.

6. Summary

In this brief part, I have described the results derived from simulations of a computable general equilibrium model. We have found that given our assumptions, the parallel market contributed the equivalent of 2 percent of the capital stock in 1977. As expected, the extent of illegal activity is most sensitive to the level of enforcement. Keeping the level of enforcement fixed, the parallel market expands very slowly as the economy grows. If enforcement is allowed to weaken, however, growth in illegal activity is much faster. A key fact which emerges from the simulations is that movements in relative prices are proportionately much smaller than movements in quantities.
Footnotes

1I am deeply indebted to Jagdish Bhagwati for many of the ideas contained in this report and to Christopher Clague, Peter Murrell and Patricia Succar for helpful suggestions. This report could not have been completed without the excellent research assistance of Kamil Yilmaz and Barbara Hopkins. I am most grateful to them.

2Terms "parallel" or "second" economy will be used in this report to refer to illegal economic activities.

3Kaiser (1976), a Western journalist, suggests that the size of the unofficial economy in the Soviet Union is 20 percent of the official one. Katz (1973) places this number anywhere between 10 and 50 percent on the basis of information provided by Soviet emigres.

4Nguyen and Whalley (1987) incorporate the parallel market into a computable general equilibrium model of the Indian economy.

5Ericson (1983, 1984) analyzes the role of parallel markets in improving the allocation of intermediate inputs among state enterprises. Starting from a suboptimal allocation of intermediate inputs across enterprises by the planning agency, he demonstrates that the presence of parallel markets yields a Pareto superior allocation of intermediate inputs.

6For example, Desai (1986) writes, "(T)he Soviet enterprises...are plagued by the Leibenstein disease of X-inefficiency..."
7 According to the evidence provided by Treml et al (1977), profits, generated mostly by state enterprises, constituted the largest single item in the state's income accounts in the year 1966. The turnover tax was the second largest item.

8 Once again, see Treml et al (1977) for empirical evidence.

9 I will not consider, however, the lobbying activities of the legal firms to secure more allocations. These types of directly-unproductive profit-seeking (DUP) activities have been considered under the rubric of "soft-budget constraints" by Kornai (1982) as discussed in Desai (1986, 1987). For a general discussion of DUP activities, see Bhagwati (1982).

10 The demand functions shown in (3) and (4) have a somewhat unconventional form. They can be derived as follows. Homotheticity implies \( \frac{G_2}{G_1} = h(p_1, p_2) \) where \( h(.) \) exhibits zero degree homogeneity and \( \frac{\partial h}{\partial p_1} \) and \( \frac{\partial h}{\partial p_2} \) are, respectively, positive and negative. The state's budget constraint is \( p_1 G_1 + p_2 G_2 = r(.)K \). Solving these two equations and defining \( g_1 = p_1 + p_2 \cdot h(.) \) and \( g_2 = \frac{p_1 + p_2 \cdot h(.)}{h(.)} \), we obtain equation (3). Equation (4) can be derived analogously.

11 Note that a similar condition for good 2, \( X_2 = G_2 + D_2 \), exists but can be dropped in view of Walras's Law.

12 It may be noted that analytically speaking, the model considered in this section may be viewed as a special case of the two-individual, capital-labor model of income distribution pioneered by Johnson (1959).

13 Enforcement could also be directed against enterprises engaging in the diversion of capital. This task is left, however, for the next section.
The reason why this assumption is necessary to ensure coexistence of state enterprises and illegal firms can be explained as follows. If $q(.)$ were assumed to depend on the firm-level output, individual firms' marginal costs will be rising in their own output. Perfect competition will force each firm to produce an infinitesimally small output and we will obtain constant average and marginal costs at the industry level. Depending on whether these costs are below or above the average costs of state enterprises, illegal firms will supply the entire or no output. The assumption made here gives rise to an external diseconomy at the firm level and hence rising average and marginal costs at the industry level. For analogous assumptions in the smuggling literature, see Bhagwati and Hansen (1973) and Sheikh (1974).

Alternative penalties such as a partial confiscation or economic equivalents of jail terms can be introduced without difficulty. But these alternatives add little to the substance of our analysis.

The welfare results, especially for workers, should be interpreted in terms of expected welfare. Ex post, the workers employed in firms not caught receive a higher wage than those employed in state enterprises and may benefit as a result of the emergence of the parallel market.

Strictly speaking, we must also assume that inspection at the time of raids is so thorough that enterprises with a small proportion of diversion are as likely to be caught as those with a high proportion. I am not entirely comfortable with this assumption but have chosen to pursue the present case because of its simplicity. The more realistic case where an enterprise can influence its probability of detection by changing the enterprise-level ratio of diverted-to-total capital is considered below.
18 Alternatively, we could work with a separate preference function for the state. As noted earlier, this will complicate the analysis without altering the broad results.

19 In the absence of a bequest, no new investment takes place in the terminal period, period 1. Therefore, the resource allocation problem in that period becomes very simple.

20 Exogenous growth in labor supply can be incorporated into the analysis in a straightforward manner. This modification will not alter any of our results.

21 Given constant returns to scale, the target for an individual enterprise can be set in proportion to its size relative to the industry wide target.

22 If the rental income is less than what is needed to cover investment costs, taxes will have to be introduced. It deserves noting, however, that at least in the Soviet case gross income generated by capital exceeds gross investment (see Gallik et al). The turnover tax is levied to cover expenditures on defense, public services and salaries of bureaucrats.

23 For details and justifications regarding the probability-of-detection function, see Part I of this report.

24 For instance, this approach will fail to take into account the effect of the parallel market in period 1 on the optimal plan in period 0.

25 This qualification is in addition to the usual qualifications with respect to the reliability of the Soviet data.

26 The presumption here is that it is the illegal labor income which does not get reported. By contrast, capital flows into the second economy through state enterprises and is included in the official accounts on which the input-output table is based.

27 Labor growth was taken from published data on the Soviet economy.
Appendix

This appendix shows that the curve $\bar{NN}'$ in Figure 6 is parallel to the curve $NN'$.

Recall that the transformation curve between $I_0$ and $C_0$ is written

(A1) $I_0 = f(C_0)$

We can rewrite this equation as

(A2) $C_0 = g(I_0)$

where $g$ is the inverse function of $f$. The output of good 1 in period 1 is

(A3) $C_1 = F(K_0 + I_0, L_0) = F(K_0 + f(C_0), L_0)$

where the second equality is obtained by using (A1). Differentiating (A3) with respect to $C_0$, we have

(A4) $dC_1/dC_0 = F_1(K_0 + I_0, L_0) f'(C_0) = F_1(K_0 + I_0, L_0) f'(g(I_0))$

where $F_1(.)$ is the partial derivative of $F(.)$ with respect to the first argument.

In the presence of the parallel market, a given $I_0$ is accompanied by a larger $C_0$ than in its absence. More precisely, in view of (A2),

(A5) $\tilde{C}_0 = [(1-x) + \frac{x}{1-\alpha}] g(I_0)$

From (A5), we can get

(A6) $I_0 = f\left(\frac{\tilde{C}_0}{(1-x) + \frac{x}{1-\alpha}}\right)$

Also, for a given $I_0$, the output of good 1 in period 1 in the presence of the parallel market is

(A7) $\tilde{C}_1 = \{(1-x) + \frac{x}{1-\alpha}\}.F(K_0 + I_0, L_0)$

Putting for $I_0$ from (A6) into (A7) and differentiating, it can be shown that $d\tilde{C}_1/d\tilde{C}_0$ is exactly the same as the last expression in (A4). Thus, for a given value of $I_0$, the slope of $\bar{NN}'$ is the same as that of $NN'$. 

56
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Figure 5
Figure 9
TABLE 1: 1977 SOVIET INPUT-OUTPUT TABLE IN PRODUCER’S PRICES: 2 SECTORS
(in million rubles, employment in thousands)

<table>
<thead>
<tr>
<th></th>
<th>IND1</th>
<th>IND2</th>
<th>Cons.</th>
<th>INV</th>
<th>GVO</th>
</tr>
</thead>
<tbody>
<tr>
<td>IND1</td>
<td>97751</td>
<td>18426</td>
<td>152762</td>
<td>78940</td>
<td>347879</td>
</tr>
<tr>
<td>IND2</td>
<td>136265</td>
<td>252253</td>
<td>75662</td>
<td>85106</td>
<td>549286</td>
</tr>
<tr>
<td>WAGES</td>
<td>58854</td>
<td>141296</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PROFITS</td>
<td>55009</td>
<td>137311</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tot.Outlay</td>
<td>347879</td>
<td>549286</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td>31435.1</td>
<td>60862.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FixCapital</td>
<td>132512</td>
<td>699018</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage/Fix.K</td>
<td>0.444</td>
<td>0.202</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage/Profi</td>
<td>1.070</td>
<td>1.029</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Final aggregation of Gallick et al I-O table without parallel market)
TABLE 2: 1977 SOVIET I-O TABLE IN PROD'S PRICES : 2 SECTORS
(in million rubles, employment in thousands)

<table>
<thead>
<tr>
<th></th>
<th>PIND1</th>
<th>PIND2</th>
<th>Cons.</th>
<th>INV</th>
<th>GVO</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIND1</td>
<td>104616</td>
<td>19020</td>
<td>169736</td>
<td>78940</td>
<td>372312</td>
</tr>
<tr>
<td>PIND2</td>
<td>145836</td>
<td>260380</td>
<td>75662</td>
<td>85106</td>
<td>566984</td>
</tr>
<tr>
<td>WAGES</td>
<td>66851</td>
<td>150273</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PROFIT</td>
<td>55009</td>
<td>137311</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tot.Outlay</td>
<td>372312</td>
<td>566984</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employment</td>
<td>31435</td>
<td>60863</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FixCapital</td>
<td>132512</td>
<td>699018</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage/Profi</td>
<td>1.215</td>
<td>1.094</td>
<td>%change inPROFITS</td>
<td>9.7282%</td>
<td></td>
</tr>
<tr>
<td>Profit/FK</td>
<td>0.415125</td>
<td>0.196434</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1-dpr)*FK</td>
<td>198201</td>
<td>714222</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Final aggregation of Gallick et al I-O table with parallel market (10% of ind1)
TABLE 3: CHANGES IN ENFORCEMENT, BRIBES AND TECHNOLOGY
(The results in this file are based on the simulations in SIMULATE.PM2)

<table>
<thead>
<tr>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>IN</td>
<td>80.89568</td>
<td>-0.77551%</td>
<td>-3.01522%</td>
<td>-2.49150%</td>
<td>-0.27660%</td>
<td>-0.81892%</td>
<td>-0.46133%</td>
<td>-4.13285%</td>
</tr>
<tr>
<td>C1</td>
<td>169.73690</td>
<td>0.71634%</td>
<td>2.78549%</td>
<td>2.30168%</td>
<td>0.25545%</td>
<td>0.75641%</td>
<td>0.66403%</td>
<td>8.94856%</td>
</tr>
<tr>
<td>W</td>
<td>1.00000</td>
<td>0.00389%</td>
<td>-0.00776%</td>
<td>0.00209%</td>
<td>-0.00196%</td>
<td>0.00185%</td>
<td>0.00399%</td>
<td>0.01537%</td>
</tr>
<tr>
<td>R</td>
<td>1.00000</td>
<td>-0.00427%</td>
<td>0.00852%</td>
<td>-0.00230%</td>
<td>0.00215%</td>
<td>-0.00203%</td>
<td>-0.00438%</td>
<td>-0.01687%</td>
</tr>
<tr>
<td>P1</td>
<td>0.99996</td>
<td>0.00009%</td>
<td>-0.00018%</td>
<td>0.00005%</td>
<td>-0.00005%</td>
<td>0.00004%</td>
<td>0.00009%</td>
<td>0.00035%</td>
</tr>
<tr>
<td>X</td>
<td>0.99998</td>
<td>13.33385%</td>
<td>50.80059%</td>
<td>5.51528%</td>
<td>42.17593%</td>
<td>4.77751%</td>
<td>14.07474%</td>
<td>10.06505%</td>
</tr>
<tr>
<td>PP</td>
<td>1.10656</td>
<td>-1.48116%</td>
<td>-5.64356%</td>
<td>-0.61264%</td>
<td>-4.68533%</td>
<td>-0.53049%</td>
<td>-1.56346%</td>
<td>-1.17777%</td>
</tr>
<tr>
<td>GX</td>
<td>0.09998</td>
<td>-13.78583%</td>
<td>-55.45643%</td>
<td>-5.63985%</td>
<td>-45.45755%</td>
<td>-4.88037%</td>
<td>-14.56711%</td>
<td>8.81847%</td>
</tr>
<tr>
<td>U</td>
<td>0.6078033</td>
<td>0.07251%</td>
<td>0.25140%</td>
<td>0.03052%</td>
<td>0.21370%</td>
<td>0.02647%</td>
<td>0.07641%</td>
<td>0.17891%</td>
</tr>
</tbody>
</table>

CASE 0: BENCHMARK EQUILIBRIUM
A = 0.88123
E = 1.50
B = 1.00
LBD = 1.00 (when LBD exceeds 1, illegal firms are more efficient than legal ones)

CASE 1: A = 1.00
CASE 2: A = 1.50
CASE 3: B = 0.90
CASE 4: B = 0.60
CASE 5: E = 2.00
CASE 6: E = 3.00
CASE 7: LBD = 1.05
CASE 8: LBD = 2.00
### TABLE 4: EVOLUTION OF THE ECONOMY FOR 4 YEARS

**AS CAPITAL AND LABOR GROW**

(The results are based on the simulations in SIMULATE.PM3 file)

<table>
<thead>
<tr>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
<th>CASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>(\text{IN} )</td>
<td>80.89</td>
<td>6.18%</td>
<td>7.15%</td>
<td>7.85%</td>
</tr>
<tr>
<td>(\text{C1} )</td>
<td>169.74</td>
<td>6.15%</td>
<td>7.12%</td>
<td>7.82%</td>
</tr>
<tr>
<td>(\text{W} )</td>
<td>1.00</td>
<td>5.52%</td>
<td>5.29%</td>
<td>4.75%</td>
</tr>
<tr>
<td>(\text{R} )</td>
<td>1.00</td>
<td>-5.79%</td>
<td>-5.56%</td>
<td>-5.01%</td>
</tr>
<tr>
<td>(\text{P1} )</td>
<td>1.00</td>
<td>0.14%</td>
<td>0.13%</td>
<td>0.12%</td>
</tr>
<tr>
<td>(\text{X} )</td>
<td>0.10</td>
<td>0.34%</td>
<td>0.33%</td>
<td>0.30%</td>
</tr>
<tr>
<td>(\text{PP1} )</td>
<td>1.11</td>
<td>0.10%</td>
<td>0.10%</td>
<td>0.08%</td>
</tr>
<tr>
<td>(\text{OX} )</td>
<td>0.10</td>
<td>0.30%</td>
<td>0.29%</td>
<td>0.26%</td>
</tr>
<tr>
<td>(\text{U} )</td>
<td>607.80</td>
<td>6.18%</td>
<td>7.15%</td>
<td>7.84%</td>
</tr>
</tbody>
</table>

**CASE 0**

**BENCHMARK EQUILIBRA = 0.88123**

\[ E = 1.50 \]
\[ B = 1.00 \]
\[ LBD = 1.00 \]

---

**CASE 1**

2nd period

**CASE 2**

3rd period

**CASE 3**

4th period

**CASE 4**

5th period
TABLE 5: EVOLUTION OF THE ECONOMY FOR 4 YEARS AS CAPITAL AND LABOR GROW, AND ENFORCEMENT BECOMES WEAKER (The results are based on simulations in SIMULATE.PM4)

<table>
<thead>
<tr>
<th>CASE</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>IN</td>
<td>80.89</td>
<td>5.36%</td>
<td>5.67%</td>
<td>5.78%</td>
<td>5.93%</td>
<td>5.97%</td>
<td>6.65%</td>
<td>7.15%</td>
<td>7.60%</td>
</tr>
<tr>
<td>C1</td>
<td>169.74</td>
<td>6.91%</td>
<td>8.40%</td>
<td>9.57%</td>
<td>10.67%</td>
<td>6.34%</td>
<td>7.56%</td>
<td>8.41%</td>
<td>9.28%</td>
</tr>
<tr>
<td>W</td>
<td>1.00</td>
<td>5.53%</td>
<td>5.25%</td>
<td>4.67%</td>
<td>4.07%</td>
<td>5.52%</td>
<td>5.28%</td>
<td>4.72%</td>
<td>4.16%</td>
</tr>
<tr>
<td>R</td>
<td>1.00</td>
<td>-5.80%</td>
<td>-5.52%</td>
<td>-4.93%</td>
<td>-4.32%</td>
<td>-5.80%</td>
<td>-5.55%</td>
<td>-4.99%</td>
<td>-4.41%</td>
</tr>
<tr>
<td>P1</td>
<td>1.00</td>
<td>0.14%</td>
<td>0.13%</td>
<td>0.12%</td>
<td>0.10%</td>
<td>0.14%</td>
<td>0.13%</td>
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<td>X</td>
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<td>PP1</td>
<td>1.11</td>
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<td>QX</td>
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<td>U</td>
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</tbody>
</table>

CASE 0 BECHMARK EQUILIBRIUM  
A = 0.88123  
E = 1.50  
B = 1.00  
LBD = 1.00

CASE 1  A = 1.00, 2nd period  
CASE 2  A = 1.10, 3rd pd.  
CASE 3  A = 1.20, 4th pd.  
CASE 4  A = 1.30, 5th pd.  
CASE 5  E = 1.85, 2nd pd.  
CASE 6  E = 2.33, 3rd pd.  
CASE 7  E = 2.63, 4th pd.  
CASE 8  E = 3.00, 5th pd.  

73