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TITLE: The Regional Economy of the Soviet Union: An Economic Modeling Study

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## EXECUTIVE SUMMARY

This report describes the construction of a multiregional econometric model of the Soviet Union. The model is composed of 91 blocks of equations: a macroeconomic block for the USSR as a whole, macroeconomic blocks for each of the 15 union republics, and 75 sectoral blocks for each republic. The sectors identified in the model are: Industry, Agriculture and Forestry, Transportation and Communications, Construction, and Trade and Other Productive Branches.

The aim of building the model is to analyze and forecast the primary indicators of spatial developments in the Soviet economy. The model was designed so that the relationships among regional, industrial, and national indicators would be simple and follow a step-by-step procedure of planning calculations. This required considering some methodological problems related to particular features of the Soviet economy. Among these are the role of the components of the final demands, the treatment of variables as endogenous or exogenous, definition of saving, interpretation of government spending, complications arising from the role of the regional aspect, and effects of the constraints on main groups of inputs.

Interdependence among the variables is determined in the model's structure by their connections with the macroeconomic block and by the feedback from the macrolevel to all sectors of the economy. In each republic block of the model, thera is an input-output table indicating technological dependence of a sector on other sectors in and outside the republic. Distributional relationships with the union budget indicate for a republic and its sectors the dependence on the development of all other republics.

In order to develop a technique for solving the econometric system, incorporating input-output tables so that the values of output and final demand are mutually adjusted and justified, a special econometric accelerator was combined with an input-output multiplier. The possibility of treating the components of final demand and gross output as exogenous was presented by the inclusion of a mechanism to capture the feedback to gross outputs from the final demands, which are both the result of current productive activity and (as a source of investment) a factor of economic growth.

The information on which the model is based consists of time series for sector, republic, and union indicators. These include observations for 1965-1980 that are either official data or estimates. In total, there are 75 republic data files and a union data file. The following indicators form the republic files: gross value of output, net value of output, capital stock, investment, investment minus depreciation for replacement, wages, profits, depreciation, depreciation for

replacement, employment, and input-output coefficients. In addition, the file for industry for each republic contains data on the following indicators: centralized investment, turnover tax, wages in the "nonproductive" sector, and republic population. The union file lists data for payments from profits, personal tax, social insurance deductions, other revenues, financing of the economy, financing of social programs, official expenditures for defense and government, and investment financed by the budget.

The forecasting results were obtained using the MIT Troll system. They are illustrated with the following indicators by republic for 1981-1985: gross social product, national income, fixed capital stock, fixed capital investment, wages, employment, population, labor productivity, capital stock-to-output ratio, and material expenditures-to-output ratio. For example, national income defined as the sum of values of net output by sector of the economy will grow over this period by 18% for the R.S.F.S.R. and 15% for the Ukraine. These republics exhibit almost equal labor productivity growth of 9-10% which is equivalent to 1.8% per annum. Our estimates indicate that the economy-wide target of 20% for labor productivity growth in the 11th Five-Year Plan (1981-1985) will be underfulfilled, and the target for fixed investment will be overfulfilled. In general, the results of our forecast, which are preliminary only, indicate that the Soviet economy still depends on extensive factors of economic growth, and substitutes for them are far beyond reach.

In addition to this report, please see two companion studies:

V.N. Bandera, "Interregional Income Transfers in the USSR From the Standpoint of the Balance of Payments."

James W. Gillula, "Major Economic Indicators for Soviet Republics: A Survey with Selected Estimates."

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# 1. INTRODUCTION

Regional analysis has always played an important role in Soviet economic planning. Relevant problems have become increasingly important since the 1965 Economic Reform, which reintroduced the vertical branch principle of management. Resource distribution is centered in the all-union branch ministries that have gained sole control over material and financial allocations for industries with all-union and union-republic subordination of enterprises. Ministries in the Soviet Union receive allocations directly from Gosplan, and the Council of Ministers oversees the distribution of the most important materials and equipment. This branch-line centralization has contributed to long-term deterioration of economic conditions in many provincial regions and, as a consequence, stimulated attempts to coordinate the branch and the territorial approaches to planning. Another factor responsible for this deterioration was the lack of coordination of growing inter-industrial dependence on local sources of labor, water supply, sewage systems, transportation, road construction, as well as land allocation and pollution problems. Furthermore, future developments in the Soviet economy may draw the attention of analysts to the regional aspects of planning. Proposals to develop and increase the role of territorial structures have been prominent in recent discussions of economic reform.

Among the most important regional indicators are those characterizing economic growth, investment, employment, productivity, population, standards of living, etc. Much effort has been devoted by Western scholars to studying regional differences in Soviet economic

growth, productivity, investment, etc. The recent publication edited by Koropeckyj and Schroeder (1981) is an example of comprehensive analysis of the development of Soviet republics and economic regions. Among others, studies on republic input-output tables (see Gillula (1977)), Soviet regional modeling (see Bond (1980)), and republic economic development (see Koropeckyj, ed. (1977) and Bandera and Melnyk (1973)) could be mentioned).

There is little need to stress the importance of the econometric modeling of Soviet regional developments. But particular features of the planned economy such as the regulation of almost all aspects of economic activity, the role of distributional mechanisms, and the close relationships between regional economic units and the state budget and other macroindicators are factors that require special methods of analysis. Though the nature of the information available on the Soviet economy is an important consideration because of numerous gaps, discrepancies, and inconsistency in the data, statistical inference proves to be fruitful due to the close interdependence among the economic indicators. An example of successful Western studies of the Soviet national economy based on this approach is the WEFA Soviet econometric model, SOVMOD (see Green and Higgins (1977)).

To some extent, the multiregional econometric model described in this report is an extension of the WEFA Soviet econometric model in the spatial dimension of the Soviet economy. But the model described here is primarily designed to be close to Soviet planning methodology itself in explaining direct and feedback connections among regional, industrial, and national indicators. The experience of the author in economic planning and econometric modeling in the Soviet Union is reflected in this study. For

example, this experience suggests that simple relationships among the indicators in a model following the step-by-step procedure of planning calculations usually operate better than complicated explanatory equations. While, for many reasons, the size of the model itself may not be too significant, the adequacy of economic information and the techniques employed are very important issues.

The purpose of this report is to present the results of building a multiregional econometric model of the Soviet Union. The fifteen Soviet republics, with five sectors of the economy represented in each, along with the national macrolevel, are the block units and subdivisions of the model. At its present stage, the model incorporates some 900 equations, relationships, and conditions.

The contents of the report are divided into chapters illustrating the methodological problems of building the model, the structure of the model, and the problem of incorporating input-output tables, the data and their sources, and some applications. The results presented, especially those concerning the structure and specification of the model, are necessarily fragmentary. A full list of the model's equations is given in the appendix.

## 2. PARTICULAR FEATURES OF A PLANNED ECONOMY AND ITS REGIONAL ASPECT

Without presenting a complete description of a planned economy, we would like to make some comments on the specific character of such an economy which needs to be taken into account in macroeconomic modeling. To some extent, there are similarities in the modeling of free-market and planned economies. In both cases, one deals with consumption, investment, production, technological constraints, prices, income distribution, etc. On the other hand, the procedures that determine the outcome of forecasts in each case are different, and not only in the interpretation of variables or political and ideological phraseology.

In a model of a free-market economy, components of final demand play the determining role. For a planned economy, the situation is different. Simplifying the matter, it could be said that not only does consumption not play a determining role, but it is doubtful that planners even know what consumers would like to buy. More attention is paid to the demands of enterprises for raw materials, energy, and equipment distributed through the system of material supply. Moreover, enterprises do not "buy" these goods, and money is used in the supply system only for accounting. Ex-ante market demand of the population in the sense of what people want to buy cannot be estimated from the data available. Instead, there is a different sort of demand -- the so-called "satisfied demand" -- which is used in planning and statistics. It is determined by the level of output and priorities for the current period and, most important, by the patterns of outputs and inputs developed in the previous period. It is clear that this ex-post demand is actually supply and, therefore, the computational sequence should be the reverse of that for a free-market economy.

Another difference stems from the treatment of variables as endogenous or exogenous. Although this is a rather controversial problem in general, there are several obvious propositions that are important in the present study. In a free-market model, money, prices, and interest are endogenously related. Equilibrium in a real goods market is impossible without monetary equilibrium, no matter what view of the monetarist-fiscalist controversy is held. In a planned economy, money supply is fixed by the state, prices are determined on the basis of normative considerations, and the rate of interest plays no role at all. Only real goods are important for the planners and economic units.

Confusion can arise also in the definition of saving, which is quite different for each of the economies. In Western macroeconomic theory, one uses this indicator as a sum of personal and business savings. Government saving exists also, but, for accounting purposes, government spending is not divided into consumption and investment goods. Within a model of a national product account, saving is exogenous with respect to investment plans because decisions to invest and save are made by different economic units. In a planned economy, there are two categories of saving -- state and personal. The former includes the government's financing of the national economy and profits of state enterprises and organizations retained for their development and decentralized investment. Only this category is counted as saving which is the counterpart of investment in macroeconomic theory. Personal saving is to be viewed merely as postponed consumption. Also important is that saving by definition equals investment because it is determined in planning at the level of capacities of the construction and installation work. In practice, however, intended

investment is different from actual saving since the construction industry and industrial suppliers fail to meet their plan targets. Therefore, in a planned economy, the macroproblem of equilibrium between investment and saving involves both macro- and microlevels of the decision-making process, with the implication of direct and feedback relationships between them.

The interpretation of government spending is another potential source of confusion. In a planned economy, government spending should not be separated from consumption and investment in national product accounting. Consumption, investment, and net exports should be the only items in the national product. Within consumption, personal and public categories can be separated, the latter equal to government spending for housing, municipal services, health, education, administration, defense, and other institutions of the "nonmaterial" sphere serving the population. And within investment, almost all will be public.

For the regional aspect of the modeling of a planned economy, distinguishing several definitions of national income is important. Of the three kinds of national income -- produced, distributed, and utilized -- the third is a crucial problem in regional planning. The first (produced) is similar by definition to the net national product measured as the net value added of enterprises producing material goods and services. The second (distributed) is close to the national income defined as a sum of incomes of state enterprises, organizations, and employees. The third (utilized) is similar to the final demands for goods and services, but in the sense defined above, i.e., "satisfied demand." For the Soviet Union as a whole, the differences among all three kinds of income are caused by losses, errors, and omissions. For Soviet republics, however, the level of

"satisfied demand" can vary, with an implication that utilized income is not equal to that produced. When income utilized in a republic is lower than that produced, the republic's product is greater than actual consumption and capital accumulation (basically, the latter equals fixed investment plus increase in inventory). This means that some part of the product is distributed elsewhere in the country without compensation to the republic. Therefore, republic investment does not necessarily depend fully on the operation of its enterprises and saving may not be equal to the value of investment goods.

In regional planning, the meaning of variables is not as obvious as for the nation as a whole. In the Soviet Union, there are three categories of enterprises and organizations -- union, union-republic, and republic -- and, according to the present economic organization, only the last belongs to the republic economy. But at the final stage of the development of a detailed draft plan, indicators of the so-called comprehensive territorial plan are computed. They cover all enterprises and organizations in the republic, regardless of their category. In the regional units of the econometric model discussed below, indicators are defined in accordance with the comprehensive territorial plan.

Such a breakdown of the economy, when some industries have direct vertical connections with the macrolevel and some interact with it only through horizontal republic institutions, is reflected in the model with two types of relationships. The first shows direct flows of payments out of profits, social insurance deductions, and others from industries to the states budget. And the second involves the intermediate territorial level for relevant indicators such as turnover tax, taxes on the population, etc.

To some extent, these relationships imitate a dual principle of management -- branch and territorial -- where the priority belongs to the former.

Informational flows of three stages of planning -- control figures, draft plan, and assignment of the targets to executors -- differ with respect to direction, purpose, content, and the degree of aggregation. The purpose of the first stage is to determine aggregate guidelines for all economic units with the use of some hypotheses, assumptions, and goals set up on the basis of the normative approach. The results of modeling can explain the main technological relationships of this stage of planning at some level of formalization and aggregation.

There are some principles of computation at the first stage of planning mentioned above that must be taken into consideration in the modeling methodology. The procedure has an iterative character due to the fact that industrial variables starting the sequence of calculations depend on the output of the system as a whole. In other words, all economic units of the system need information as to the limits of resources that can be allocated to them. The reason for this iterative organization of calculations is that the approximate output of the system is to be found first. Only then can the resources resulting from it be distributed, and the technological sequence of calculations start. Further, the initial version of output is corrected, and, on the next iteration, changes are made in the values of all indicators in the same order as on the first iteration. In the model, this procedure is reflected in a set of feedbacks from the state budget variables, itemizing the financing of the national economy, to the indicators of industrial development.

Thus, to start calculations at the stage of control figures, initial constraints for major groups of resources -- labor, investment, and raw materials -- must be known. Ordering and distributing material inputs in physical terms is a perpetual process which starts before and ends after the plan of production is confirmed. For this reason, constraints on material inputs are practically not taken into account at the control figures stage, which causes numerous complications and corrections at other stages. What industries and regional units have to know at this stage is their limits on labor force and investment. While constraints for the former are determined precisely enough at the republic level (with the use of horizontal links), the latter is distribute strictly on the vertical principle. Only when ministries, departments, and republics acquire information on these limits, can they start compiling their plan targets. This iterative process is interpreted in the model with simultaneous calculations of indicators at macro- and microlevels of the economy.

## 3. THE STRUCTURE OF THE MODEL

The general structure of the model consists of 75 republic sector blocks, 15 republic blocks, and a macroeconomic block. The republic blocks are as follows:

1. Armenia
2. Azerbaidzhan
3. Belorussia
4. Estonia

5. Georgia
6. Kazakhstan
7. Kirgizia
8. Latvia
9. Lithuania
10. Moldavia
11. R.S.F.S.R.
12. Tadzhikistan
13. Turkmenistan
14. Ukraine
15. Uzbekistan

In each of these republics there are five sector blocks identified as follows:

1. Industry
2. Agriculture and Forestry
3. Transportation and Communications
4. Construction
5. Trade and Other Productive Branches

Interdependence among the variables is determined in the model by their connections with the macroeconomic block and by the feedback from the macrolevel to all sectors of the economy (see Figure 1). In each republic block of the model, there is an input-output table indicating technological dependence of a sector on other sectors in and outside the republic. Distributional relationships with the union budget indicate for a republic and its sectors the dependence on the development of all other republics.

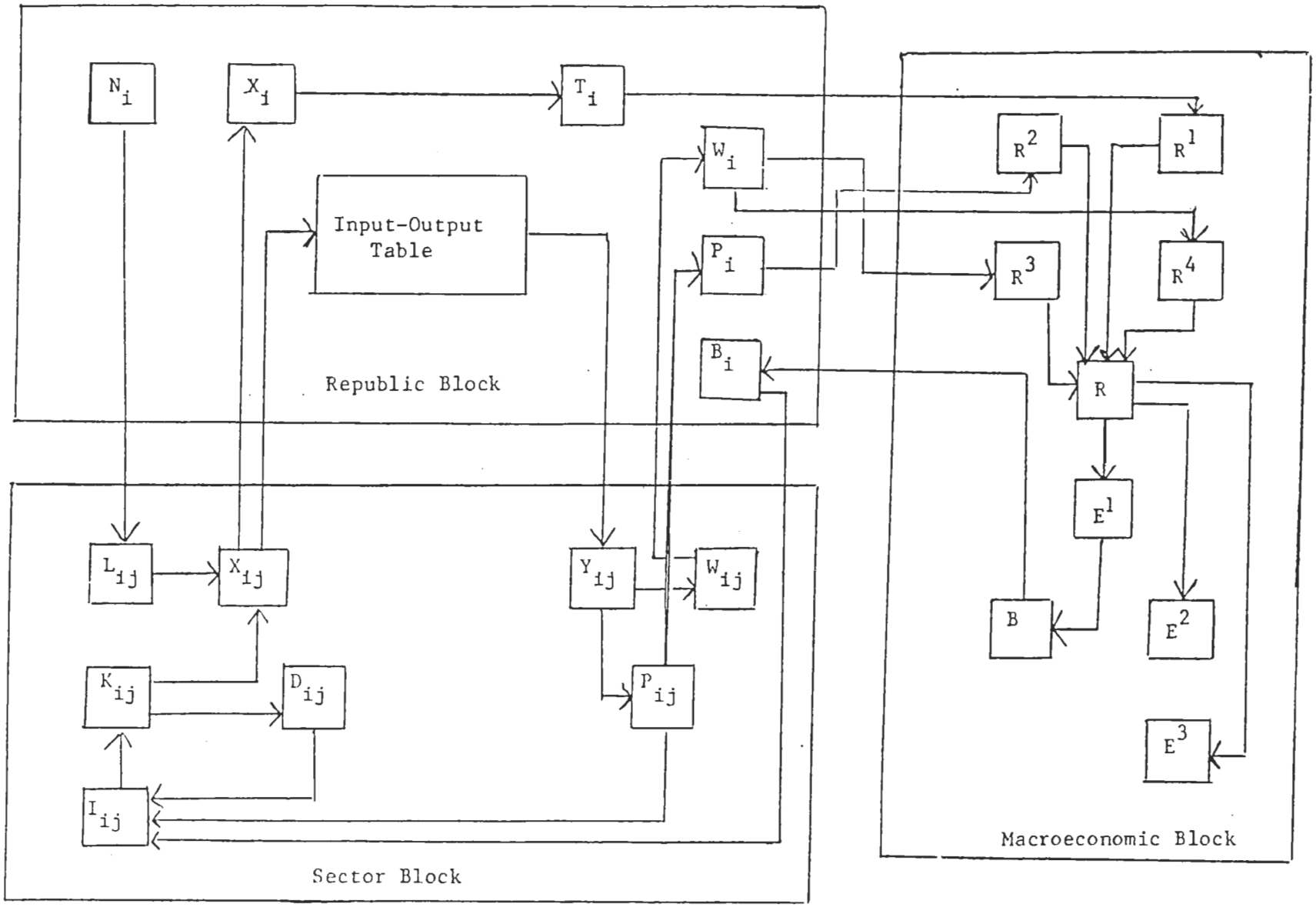


Figure 1 Structural Relationships among the Sector, Republic, and Macroeconomic Blocks

Figure 1 illustrates some of the relationships among the indicators of the sector, republic, and macroeconomic blocks. The notation of variables in Figure 1 is as follows:

X = gross value of output;

Y = net value of output;

K = capital stock;

I = investment;

W = wages;

P = profits;

D = depreciation;

L = labor;

N = population;

T = turnover tax;

$R^1$ = total turnover tax;

$R^2$ = payments from profits;

$R^3$ = taxes on population;

$R^4$ = social insurance deductions;

R = budget revenue;

$E^1$ = financing of the national economy;

$E^2$ = financing of social and cultural programs;

$E^3$ = expenditure for defense and government;

B = investment component of budget.

Some of the main relationships among the indicators starting from the macroeconomic block are treated below in more detail.

The first budget revenue item is the sum of turnover taxes paid by republics as territorial units of the economy and levied on consumer goods,

usually at the initial stage of their production. It is computed in two sectors of the economy: industry, and trade and distribution. But indeed all sectors participate in the production of consumer goods, either directly, e.g., agriculture, or indirectly, e.g., construction and transportation. For this reason, the aggregate republic turnover tax function is defined as depending on the output of all five "productive" sectors of the economy*:

$$R^1 = [\sum_{i=1}^{15} \nu_{i0} + \nu_{i1}(\sum_{j=1}^{5} X_{ij})],$$

where $X_{ij}$ = gross value of output in $j^{th}$ sector of $i^{th}$ republic. The expression in brackets is turnover tax "produced" in $i^{th}$ republic, and it differs for each republic by the values of coefficients $\nu_{i0}$ and $\nu_{i1}$. These coefficients are estimated from Soviet statistics and vary from republic to republic depending on the structure of their outputs. A republic with a higher share of consumer goods produced will have a relatively greater estimate of $\nu_1$. The composition of both estimates -- $\nu_{i0}$ and $\nu_{i1}$ -- reflects the trend in the change of turnover tax in the $i^{th}$ republic with respect to the growth of its output and indicates to what extent such a growth is favorable for consumer goods. For example, other things unchanged, the higher $\nu_{i1}$ and lower $\nu_{i0}$, the greater is the gradual shift to the production of consumer goods in the $i^{th}$ republic.

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*Since we do not discuss statistical problems of estimation, the residual terms in all equations are omitted. Here and below, Greek letters denote estimated coefficients. In an equation for the USSR as a whole, they have one subscript and, for a republic and republic sector of the economy, two and three subscripts, respectively.

The second budget-revenue item, payments from profits, consisting of payments for capital stock, circulating capital, and the free residual, depends on the total value of profits earned by enterprises of all the republics:

$$R^2 = \rho_0 + \rho_1 \left( \sum_{i=1}^{15} P_i \right),$$

where $P_i$ = total amount of profit made by the enterprises located in the territory of $i^{th}$ republic, regardless of the level of their subordination. Profits for all republics are calculated in advance, and then the function for payments from profits is defined.

Population-taxes and social insurance-deduction components of budget revenues are defined as functions of the total wage bill:

$$R^k = \gamma_0^k + \gamma_1^k \left( \sum_{i=1}^{15} W_i \right), \quad k = 3,4,$$

where $W_i$ = wages paid by the enterprises and organizations of the $i^{th}$ republic calculated in each of the above two functions.

Taking into account the fifth budget-revenue item, the miscellaneous part of budget revenues, defined as a linear function of all other items totalled previously, yields the following equality for budget revenues:

$$R = \sum_{k=1}^{5} R^k.$$

All three components of expenditures (financing of the national economy, financing of social programs, and expenditures for defense and government) are modeled as functions of budget revenues:

$$E^k = \tau^k_0 + \tau^k_1 R, \quad k = 1,2,3.$$

To maintain the feedback to industrial indicators, the investment component (B) is isolated in the expenditures for the development of the national economy:

$$B = \sigma_0 + \sigma_1 E^1.$$

At this stage, budget indicators are prepared for linking with the main regional indicators. From the standpoint of industrial units of republic models, budget expenditures are exogenous inputs, and, from the standpoint of the budget balance, industrial inputs are exogenous. But for the closed model, both kinds of inputs are endogenous. Centralized investment is explained as flowing from the budget to the republic level of the economy:

$$B_i = \delta_{i0} + \delta_{i1} B, \quad i = 1,\ldots,15,$$

where $B_i$ = centralized investment at the republic level of the economy, so that the balancing equality is $B = \Sigma B_i$. Each republic's portion of total centralized investment in the economy is arrived at through the variety of different coefficients $\delta_{i0}$ and $\delta_{i1}$.

The industrial indicator of investment can be determined as a sum of three components -- centralized investment defined above, decentralized investment financed by enterprises with a statutory portion of profits retained for their productive and social development, and depreciation payments to the degree foreseen for the replacement of worn-out capital:

$$I_{ij} = n_{ij0} + n_{ij1}D_{ij} + n_{ij2}P_{ij} + n_{ij3}B_i, \quad i = 1,\ldots,15; \quad j = 1,\ldots5,$$

where $I_{ij}$, $D_{ij}$, and $P_{ij}$ = total investment, depreciation payments, and profits, respectively, in the $j^{th}$ "productive" sector of the $i^{th}$ republic. Coefficients differ for all republics and sectors conforming to different values of investment as a regional sectoral variable. Depreciation payments by sector of the economy are a function of existing capital stock:

$$D_{1j} = \lambda_{ij0} + \lambda_{ij1}K_{ij}, \quad i = 1,\ldots,15; \quad j = 1,\ldots,5,$$

and profit is a residual:

$$P_{ij} = Y_{ij} - W_{ij}, \quad i = 1,\ldots,15; \quad j = 1,\ldots,5,$$

where $K_{ij}$, $Y_{ij}$, and $W_{ij}$ = capital stock, net product, and wages, respectively, in the $j^{th}$ sector of the $i^{th}$ republic.

With the exception of agriculture, the production functions are identified in the following form:

$$X_{ij} = \alpha_{ij0} K_{ij}^{\alpha_{ij1}} L_{ij}^{\alpha_{ij2}}, \quad i = 1,\ldots,15; \quad j = 1,3,4,5,$$

where $X_{ij}$, $K_{ij}$, and $L_{ij}$ = gross value of output, capital stock, and employment, respectively, in the $j^{th}$ sector of the $i^{th}$ republic. In addition to the above factors, the agricultural production function (j=2) includes the total area sown in a republic. In total, for 15 republics and 5 sectors of the economy, there are 75 production functions of the same form but with different coefficients, as is the case for all other sectoral republic variables.

In planning theory and practice, the average annual capital stock is computed on the basis of balance considerations taking into account capital stock in the preceding year, capital put into use and scrapped, and averaging coefficients transforming the last two items into a capital-year measure. Capital put into use, in turn, depends on investment, so that the corresponding synthesized relationship between capital stock and investment is:

$$K_{ij} = K_{ij,t-1}^{+\beta_{ij0}+\beta_{ij1} I_{ij}+\beta_{ij2} I_{ij,t-1}+\beta_{ij3} I_{ij,t-2}}, \qquad \begin{matrix} i = 1,\ldots,15; \\ j = 1,\ldots,5 \end{matrix}$$

where $I_{ij}$, $I_{ij,t-1}$, and $I_{ij,t-2}$ = investment in the $j^{th}$ sector of the $i^{th}$ republic, in years t, t-1, and t-2, respectively.*

Employment functions are defined through the number of able-bodied and total republic population which are the only exogenous variables of the model, for the reason that we have available projections of FDAD of the U.S. Department of Commerce (1979). We do not consider here these and some other variables which are not used in systematic calculations.

Net industrial product is determined in the model with the use of input-output tables, according to the methodology of calculation of material expenditures:

$$Y_{ij} = (1 - \sum_{k=1}^{5} a_{ikj}) X_{ij} - D_{ij}, \quad i = 1,\ldots,15; \; j = 1,\ldots,5,$$

where $a_{ikj}$ = technological coefficient of expenditures of the $k^{th}$ sector's product in the production process of the $j^{th}$ sector in the $i^{th}$ republic. In total, there are 15 input-output matrices $(a_{ikj}, i=1,\ldots,15)$, one for each of the republics.

The scheme described above reflects closed technological and distributional relationships among the indicators at macro and microlevels

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*Such a number of years is chosen because three years is often considered in planning procedure as an average period of industrial construction. But even if, most likely, it is longer than that, the final years of construction are the most important in the sense of most intensive investment.

of the economy. These relationships are based on the double role of the net national product discussed in more detail in the author's publication with Emel'ianov (1974). This indicator is the result of the current productive activity and, at the same time, a factor influencing economic growth in the current and future cycles of production.

## 4. INPUT-OUTPUT TABLES WITHIN THE ECONOMETRIC MODEL

Although, as alternative techniques, input-output and econometric models are usually developed in parallel both for free-market and planned economies, some writers have used input-output tables and their components in econometric research. The pioneering attempt was made by Fisher, Klein, and Shinkai (1965) who initially computed final demands series with input-output relations and then applied regression techniques to explain them by relevant GNP components. Among the further attempts is an interesting idea developed by Green, Guill, Levine, and Miovic (1976). They included total material inputs as a third factor, along with labor and capital, in the estimated production functions of the SRI-WEFA Soviet Econometric Model, finding these inputs as column totals of the input-output flow matrix which is revised at each iteration.

Trying to circumvent the problem of scarcity of input-output data, some writers proceeded to use either "direct" methods such as RAS, or "indirect" econometric techniques of estimating their changes over time. Thus, Preston (1975) used the econometric approach, developed by Hickman

and Lau (1973) for the international trade share matrix, in estimating input-output flows within the Wharton Annual Model. The same procedure, with a slight difference in detail, was applied by Guill (1977) for the SRI-WEFA Soviet Econometric Model. The approach developed illustrates how input-output and econometric techniques can be mutually helpful, but, from the standpoint of applications, it looks rather complicated. While the target is total industrial inputs, their components -- input-output flows -- are to be computed beforehand. It is not obvious, however, that it is easier to find element-by-element change of an industrial input vector than to build a procedure that can explain such change as a whole.

The purpose of this section is to illustrate another principle of interaction of input-output and econometric systems based on results which the author previously reported in 1976, going back to the Leontief (1953) dynamic system:

$$X - AX - B\dot{X} = Y, \qquad (1)$$

where X= nxl vector of gross outputs (the dot over X denotes its rate of growth), A= nxn matrix of material input coefficients, B= nxn matrix of capital input coefficients, and Y = nxl vector of final demands.

To find the general solution of this non-homogeneous system of n linear differential equations, it is necessary to postulate an autonomous functional form of the vector function Y(t). Theoretically, we might do it but, practically, as Leontief says (1953, page 65): "The final bill of goods described by functions $Y_1,\ldots,Y_n$ can, strictly speaking, comprise

any demand not derivable, i.e. not explainable, on the basis of the structural input-output relationships explicitly accounted for by the flow and stock coefficients appearing on the left-hand side of the particular open system used."

In order to develop the technique for solving system (1) so that values of X and Y are mutually adjusted and justified, we will isolate the built-in differential accelerator, replacing it with a special econometric system, and retain the multiplier effect for the input-output system. A more adequate description of the feedback to the gross outputs from the final demands, which is both the result of current productive activity and (as a source of investment) a factor of economic growth, will make it possible to treat the components of vector Y, along with X, as endogenous.

To illustrate the origin of the econometric accelerator in the model, we will use, besides X and Y, the following four nx1 variables: K = capital stock, I = investment, D = depreciation, K = employment; and three scalar variables: Z = national income; S = saving of state enterprises and organizations; N = population. The general specification of its equations can be written as the following simplified version of the model described above:

1. Gross output $X_{it} = X_i(K_{it}, L_{it})$, $i=1,\ldots,n$

2. Capital stock $K_{it} = K_i(K_{it-1}, I_{it}, I_{it-1}\ldots)$, $i=1,\ldots,n$

3. Investment $I_{it} = I_i(S_t, D_{it})$, $i=1,\ldots,n$

4. Depreciation $D_i = D_i(K_{it}), \quad i=1,\ldots,n$

5. Employment $L_{it} = L_i(N_t), \; i=1,\ldots,n$

6. Accumulation $S_t = S(Z_t)$

7. National income $Z_t = \sum_{i=1}^{n} (Y_{it} - D_{it})$

8. Population $(N_t = N(t).$

These are the equations for the jth sector of the economy for the ith republic which are used in computations for the system as a whole. The structure of the model follows the distributional character of Soviet economic planning so that some policy indicators could play a direct regulator role for the republics and sectors of the economy. In a planned economy, of course, any of the indicators may be used as a policy instrument, but in the planning process the three basic groups of indicators are labor, investment, and the supply of resources. At the initial stage of planning ("control figures") outlining the main guidelines for the development of the economy, the first two are most important, and they are included in the model.

The constraints for employment are determined precisely enough at the republic level, but the bulk of investment, i.e., its centralized part, is distributed strictly on the vertical principle. That determines the iterative character of planning at the "control figures" stage. To put it differently, the entire sequence of calculations depends on the output of

the system as a whole which, in turn, is a function of the operations of all its individual units.

In the above system, there are 5n+3 equations with 6n+3 variables (excluding t and lagged components), so that, at each step of calculation, we can find 5n+3 variables being given n arbitrary variables. Let them be the elements of vector Y; then, being restricted to the linear form of the above expressions and eliminating the intermediate variables, we will obtain the following system of equations in X, Y, and t:

$$X_t = \ell^0(t) + \ell^1 i' Y_t, \tag{2}$$

where $\ell^0(t) \equiv (\lambda_1^0(t),\ldots,\lambda_n^0(t))$; $\ell^1 \equiv (\lambda_1^1,\ldots,\lambda_n^1)$; i = nxl unity vector.

Vector function $\ell^0(t)$ separates for each industry the influence of predetermined factors such as demographic changes, prehistory of current growth, etc., and has a cumulative effect on the gross outputs. Vector $\ell^1$ is comprised of constant terms and plays the role of marginal output, resulting from the change in final demand as a factor of economic growth.

If we combine system (2), which describe a matrix accelerator, with the static form of system (1), which describes a matrix accelerator, we will obtain the expanded system of 2n equations with 2n unknown components of X and Y. Denoting the equilibrium solution by $\bar{X}$ and $\bar{Y}$, the condition of equilibrium can be written:

$$\bar{X}_t = \ell^0(t) + R\bar{Y}_t = (I-A)^{-1}\bar{Y}_t, \tag{3}$$

where $R \equiv \ell^{1} i'$, and $A \equiv (\alpha_{ij};\ i,j=1,\ldots,n)$.

System (3) performs endogenous calculations of final demands and gross outputs taking into account the structural parameters and accumulated potential of economic growth. The computing advantage of such a description is in the dimension of the dynamic system which does not depend on the extension of the lagged variables and exogenous functions, i.e., remains the same as in a static situation. All relevant complications are absorbed by the vector $\ell^{0}(t)$ which is to be recalculated at the beginning of each successive iteration.

Since $[(I-A)^{-1}-R]^{-1} = (I-A)\,[I-R(I-A)]^{-1}$, it follows from system (3) that:

$$\bar{X}_t = B^{-1}\ell^{0}(t) \text{ and } \bar{Y}_t = (I-A)B^{-1}\ell^{0}(t), \tag{4}$$

where $B \equiv I - R(I-A)$.

Components of vectors $\bar{X}_t$ and $\bar{Y}_t$ determined with expressions (4) provide the equilibrium solution of system (3). The crucial property of matrix B is that the solution can be found with immediate formulas without inverting it. The easiest way to show this is to substitute function (2) for $\bar{X}_t$ in the input-output equations summing them:

$$i'\bar{Y}_t = \gamma i'(I-A)\ell^{0}(t), \tag{5}$$

and expression (5) for $i'\bar{Y}_t$ in the econometric part of system (3):

$$\bar{X}_t = (I+G)\ell^{0}(t) \text{ and } \bar{Y}_t = (I-A)\,(I+G)\ell^{0}(t), \tag{6}$$

where: $\gamma \equiv [1-i'(I-A)\ell^1)^{-1}$ is a scalar and $G \equiv \gamma R(I-A)$.

Comparing expressions (4) and (6) yields:

$$B^{-1} = I+G$$

Giving a priori expressions of $\bar{X}_t$ and $\bar{Y}_t$ by means of the combinations of vectors $\ell^0(t)$, $\ell^1$, and matrix A, formulas (6) are very convenient in solving system (3).

To analyze the decision problem of system (3), we will consider, analogous to the usual input-output definition matrix R(I-A) productive if the following system:

$$[I-R(I-A)]\bar{X}_t > 0, \quad \bar{X}_t > 0$$

is compatible. The Brouwer-Solow sufficient criterion of productivity formulated in terms of the sum of matrix coefficients is valid for this system. But, in this case, it is possible to indicate specific conditions which are both sufficient and necessary. With this purpose, we notice that, at arbitrary positive $\ell^0(t)$, vector $\bar{X}_t$ is positive if and only if matrix (I+G) is positive semidefinite, i.e., the following inequalities are correct:

for the diagonal elements of matrix G

$$\frac{\lambda^1_j(1-\sum_i a_{ij})}{1-\sum\lambda^1_j(1-\sum_i a_{ij})} > -1, \quad j=1,\ldots,n, \tag{7}$$

for the rest of its n(n-1) elements

$$\frac{\lambda^1_j(1-\Sigma\alpha_{ik})}{1-\Sigma\lambda^1_j 1-\Sigma\alpha_{ij})} \geq 0,\ j,k = 1,\ldots,n;\ j\neq k, \tag{8}$$

and for the row totals of matrix G

$$\frac{\lambda^1_j(n-\Sigma\Sigma\alpha_{ki\ ik})}{1-\Sigma\lambda^1_j(1-\Sigma\alpha_{ij})} > -1,\ j=1,\ldots,n. \tag{9}$$

Analyzing inequalities (7)-(9), it is possible to show that they are equivalent to the following (n+1) conditions:

$$\sum_{i=1}^{n}\alpha_{ij} < 1,\quad j=1,\ldots,n, \tag{10}$$

$$\sum_{j=1}^{n}\lambda^1_j\left(1-\sum_{i=1}^{n}\alpha_{ij}\right) < 1, \tag{11}$$

which, therefore, are necessary and sufficient for the productivity of matrix R(I-A).

Inequalities (10) are nothing more than a usual input-output requirement reflecting the fact that the cost of material inputs must not exceed the value of an entire product. The left-hand side of condition (11), which is the trace of matrix R(I-A), can be viewed as the power of feedback from the results of the current productive activity to the gross outputs. Practically, it is by far less than unity because such feedback is small relative to the effect of the accumulated potential of economic growth (vector $\ell^0(t)$).

The feature of matrix R(I-A), which makes possible the above results, is its special structure: each element $\lambda_j^1(1-\Sigma_i \alpha_{jk})$, $j,k = 1,\ldots,n$, is a product of two numbers, the first of which is the same for all columns, and the second the same for all rows. Therefore, all its principal minors but the first-order singular, the rank of the matrix is one, and so is the number of its non-zero characteristic roots. Finally, this single characteristic root is equal to the trace of matrix R(I-A) and, according to condition (11), less than unity. This property ensures the $[R(I-A)]^n \to 0$ when $n \to \infty$ so that $B^{-1}$ can be expanded in a series:

$$\sum_{n=0}^{\infty} [R(I-A)] = I + R(I-A) + [R(I-A)] + \ldots,$$

and hence:

$$\bar{X}_t = \ell^0(t) + R(I-A)\ell^0(t) + [R(I-A)]^2\ell^0(t) + \ldots$$

The illustrated principle of expansion of vector $\bar{X}_t$ differs from that for a usual input-output multiplier which, of course, is also available in this case. In the familiar version, only the vector of material inputs is expanded, and the vector of final demands is included as a whole. In the developed model, the part of industrial grow output which depends on the current productive activity is distributed in successive cycles of feedback effects on economic growth. Moreover, analogous separate expansions are possible both for the vectors of material inputs and final demands.

## 5. THE DATA AND THEIR SOURCES

The time series for sector, republic, and union indicators include observations for 1965-1980, which are either official data or estimates. In total, there are seventy five republic data files, five for each of fifteen republics, and a union data file. The following indicators form republic files: $X_{ij}$ = gross value of output; $Y_{ij}$ = net value of output; $K_{ij}$ = capital stock; $I_{ij}$ = investment; $II_{ij}$ = investment minus depreciation for replacement; $W_{ij}$ = wages; $P_{ij}$ = profits; $D_{ij}$ = depreciation; $DD_{ij}$ = depreciation for replacement; $L_{ij}$ = employment; $Z_{ij}$ = input-output coefficients, where i=1,...,15 and j=1,...,5 represent the $j^{th}$ sector of the $i^{th}$ republic. In addition, the file for industry for each republic contains the data on the following indicators: $B_i$ = centralized investment; $T_i$ = turnover tax; $U_{i2}$ = wages in the "nonproductive" sector; $N_i$ = republic population. The union file lists the data for the indicators as follows: $R^2$ = payments from profits; $R^3$ = personal tax; $R^4$ = social insurance deductions; $R^5$ = other revenues; $E^1$ = financing of the economy; $E^2$ = financing of social programs; $E^3$ = expenditures for defense and government; B = investment financed by the budget.

The following is a more detailed description of the methodology used to derive the data and the sources of the data.

### Gross Value of Output (x) (Million Rubles, 1973 prices)

Whenever available, the data were taken from the yearbooks of the corresponding republics. The following data were obtained by interpolation:

Armenian - "construction" and "transportation and communications" (1966-1969, 1974), "trade and other" (1974), "republic total" (1967-1969);
Azerbaidzhan - "transportation and communications", "construction", "trade and other" (1974, 1976-1978);
Belorussia - "agriculture" (1966-1967), "transportation and communications", "construction", "trade and other" (1966-1969, 1973);
Estonia - "transportation and communications", "construction", "trade and other" (1966-1967, 1972-1974);
Georgia - "transportation and communications", "construction", "trade and other" (1966-1969, 1974);
Kazakhstan - "agriculture" (1966-1967), "transportation and communications", "trade and other" (1972-1974);
Kirgizia - "agriculture" (1966-1967), "transportation and communications", "construction", "trade and other " (1966-1969, 1974);
Latvia and Moldavia - "transportation and communications", "construction", "trade and other " (1966-1969);
Lithuania - "agriculture" (1966-1969);
R.S.F.S.R. and Uzbekistan - "agriculture" (1966-1967), "transportation and communications", "construction", "trade and other" (1966-1969);
Tadzhikistan - "agriculture" (1966-1967, 1979), "transportation and communications", "construction", trade and other " (1966-1969, 1971, 1973-1974);
Turkmenistan and Ukraine - "transportation and communications", "construction", "trade and other" (1966-1969, 1971-1974); Turkmenia - "agriculture" (1966-1967).

The following data were obtained by extrapolation using the average differences of the previous four years: (1) 1978-1980 for "construction", "transportation and communications", "trade and other" in the following republics: Belorussia, Georgia, Lithuania, Moldavia, Turkmenistan; (2) 1979-1980 for the same sectors in the following republics: Armenia, Estonia, Latvia, Tadzhikistan, Uzbekistan; (3) 1980 for the same sectors in Azerbaidzhan; (4) 1977-1980 for the same sectors in Kazakhstan; (5) 1976-1980 for the same sectors in Kirgizia; (6) 1974-1980 for the same sectors in R.S.F.S.R.

National Income (Y) (Million Rubles, 1973 prices)

All data (except those for 1975) are derived proportional to the gross value of output (GVO) for the following republics: Armenia, Belorussia, Estonia, Kazakhstan, Moldavia, R.S.F.S.R, Turkmenistan, and Tadzhikistan.

All data (except 1973) for Azerbaidzhan are taken proportional to GVO. The following data are taken proportional to GVO (all sectors): Georgia (1966-1969, 1976-1980), Latvia (1966-1969, 1974-1980), Lithuania (1966-1969, 1974, 1979-1980), Kirgizia (1966-1969, 1971-1974, 1977-1980), Ukraine (1966-1969, 1974), Uzbekistan (1966-1969, 1979, 1980).

Productive Capital Stock (K) (Million Rubles, 1973 prices)

The source for 1965-1977 data (all republics, all sectors) is "Fixed Capital in Soviet Republics in 1973 Prices", by Gillula (1981). The

1979-1980 data (all sectors) were found by extrapolation using the average increase of the previous three years for the following republics: Azerbaidzhan, Estonia, Georgia, R.S.F.S.R, Ukraine, Uzbekistan.

The 1978-1980 data (all sectors) were found by extrapolation using the average increase of the last three years for the following republics: Tadzhikistan, Turkmenistan.

Capital Investment (I) (Million Rubles, 1973 prices)

The data for "trade and other" were found by taking 10% of the sum of ("trade and other" and "non-productive sector") for all republics except: Georgia, Kazakhstan, Latvia, Lithuania.

For these republics, the 1965, 1970, and 1975 data are taken from yearbooks and for the intermediate years are interpolated. The data for "centralized investment" (1976-1980) are taken as a percentage of "total investment". The percentage used is approximately equal to the proportions of the previous years (all republics).

The data for years 1965-1975 ("industry", "agriculture", "transportation and communications", "construction", "total") are taken from SOVMOD statistics for all republics.

The following data were found by extrapolation using the average difference of four previous years: Armenia (1979-1980), Azerbaidzhan (1980, except "agriculture"), Estonia (1980), Georgia (1980), Kazakhstan (1979-1980), Kirgizia (1980), Latvia (1980), Moldavia (1980), R.S.F.S.R. (1980), Tadzhikistan (1979-1980), Turkmenia (1980), Uzbekistan (1980).

The data for years 1976-1979(80) are taken from the yearbooks of the corresponding republics.

Labor (L) (1000 people)

The 1965-1975 data (all republics) were taken from SOVMOD for the following sectors: "industry", "agriculture", "nonproductive".

The 1976-1980 data (all republics) for "industry" are taken as estimated by J. Gillula. The following data were found by extrapolation based on the average differences over the last four years: Armenia (1979-1980, all sectors except "industry"), Azerbaidzhan (1980), Belorussia (1976-1980, all sectors except "industry"), Kazakhstan (1979-1980: "total employment", "trade and other", "transportation and communications", "nonproductive"), Kirgizia (1980, all sectors), Lithuania (1979-1980: "nonproductive", total employment"), Moldavia (all sectors except "industry"), R.S.F.S.R. (1980: "transportation and communications", "trade and other", "nonproductive", "total employment"), Tadzhikistan ("trade and other", "transportation and communications", "nonproductive").

The following data were found by interpolation: Belorussia (1966-1969, 1971-1974: "transportation and communications", "trade and other"), R.S.F.S.R. (1976: all sectors except "industry", "construction"), Ukraine (1966-1969, 1971-1974: "transportation and communications", trade and other").

The 1965-1969 data for Turkmenistan ("transportation and communications", "construction", "trade and other") were derived from trend regressions using the 1970-1975 data.

Wages (W) (Million Rubles, current prices)

The 1965-1975 data for all sectors except "nonproductive sector" were found by multiplying: (average monthly wage x 12 x (employment). The data for "nonproductive sector" were found as the difference between the ("total") and (sum of all other sectors).

The 1976-1980 data were found with the use of trend regressions for the following republics; Armenia, Azerbaidzhan, Georgia, Kazakhstan (all sectors).

The 1976 data for Belorussia were found by interpolation (all sectors). The 1980 data for Estonia, Kirgizia, Latvia, R.S.F.S.R, Turkmenistan, Uzbekistan (all sectors) are based on the extrapolation of average increments in the previous four years.

The 1977-1980 data for Lithuania and Moldavia (all sectors) were found with trend regressions. The 1979-1980 data for Tadzhikistan (all sectors) were found by extrapolation based on the average increase for the past three years.

Profits (P) (Millions Rubles, current prices)

The following data (all sectors) were found with the use of trend regressions: Armenia (1979-1980), Azerbaidzhan (1980), Belorussia (1980), Estonia (1980), Georgia (1978, 1980), Kazakhstan (1965-1969, 1980), Kirgizia (1965-1969, 1980), Latvia (1980), Lithuania (1979-1980), Moldavia (1979-1980), R.S.F.S.R. (1980), Tadzhikistan (1976-1980), Turkmenistan

(1980), Uzbekistan (1980). All other data are taken from the corresponding yearbooks.

Depreciation, Total (D) and for Replacement (DD)

(Million Rubles, current prices)

The data (1965-1975: all sectors) were based on the depreciation - to - capital stock ratio from union totals for the following republics; Belorussia, Estonia, Kazakhstan. The 1976-1980 data for these republics were found with trend regressions.

The following data were interpolated:

Armenia (1966-1969: all sectors);

Azerbaidzhan (1966-1969, 1971-1974: depreciation for replacement only, and "trade and other" from 1966 to 1969 and 1971 to 1972);

Kirgizia ("trade and other" from 1966 to 1969); Lithuania (1966-1969: all sectors);

Moldavia (1973-1974: all sectors and "trade and other" from 1971 to 1974);

R.S.F.S.R. ("trade and other" from 1971 to 1974 and 1978 data for all sectors);

Tadzhikistan (all sectors from 1966 to 1969);

Turkmenistan (all sectors from 1966-1969 and 1973 to 1974);

Ukraine ("trade and other" 1966-1969; 1975 data are based on an average increase in 1973 and 1974 for each sector);

Uzbekistan (all sectors: 1966-1969, 1971-1972, and 1976).

The following data were obtained with the use of trend regressions:

Armenia ("trade and other" from 1979 to 1980);

Georgia (all sectors from 1970 to 1980);

Kirgizia (all sectors: 1965 to 1969 and 1975-1980, regressions run on data from 1970 to 1974);

Lithuania (all sectors from 1965 to 1969, with the 1970-1978 data used to obtain coefficients);

Moldavia ("trade and other" from 1979 to 1980);

Tadzhikistan (all sectors from 1974 to 1980, with the 1965-1973 data used to obtain coefficients);

Ukraine (all sectors from 1965 to 1976 and 1978-1980).

The following data were extrapolated using an average increase for the last three years: 1979 and 1980 figures for all sectors except "trade and other" for the following republics; Armenia, Latvia, and Lithuania (including "trade and other"), Moldavia, Turkmenistan (including "trade and other" from 1978 to 1980).

Figures for 1980 were extrapolated for the following republics; Azerbaidzhan, R.S.F.S.R., Uzbekistan ("trade and other" were extrapolated for 1979).

Turnover Tax (T) (Million Rubles, current prices)

The data for 1966 were derived from sources provided by the Foreign Demographic Analysis Division of U.S. Department of Commerce. The data for years 1965, 1967-1975 were found by multiplying the 1966 data by "Rates of Growth of Gross Output" (Narkhoz). Data for 1976-1980 were found by trend regressions run on 1965-1975 data.

Total Area Sown (Q) (Million hectares)

The data for republics were derived from Narkhoz SSSR, 1980.

Difference Between Capital Investment and Depreciation for Replacement (II) (Million Rubles)

The data (for all republics and all sectors) were found by subtracting the values for Depreciation for Replacement (DD) from those for Capital Investment (I).

Budget Data (Million Rubles, current prices)

Centralized Investment(B) data for 1965-1975 were found as the sum of two types of decentralized investment listed in the "Consolidated State Budget for the Union as a whole and for the Republics" handbook: decentralized capital investment and decentralized capital investment of organizations and enterprises. The data for Payment for Profits ($R^2$), Personal Tax ($R^3$), Social Insurance Deductions ($R^4$), Other Revenues ($R^5$), Financing the Economy ($E^1$), Financing of Cultural Measures ($E^2$), Defense and Government ($E^3$) were taken directly from Narkhoz tables for all budget indicators.

## 6. ILLUSTRATION OF THE FORECASTING RESULTS

Table 2 illustrates the projected growth of ten main macroindicators by republic for 1981-1985 calculated with the use of the model described above. The indicators are as follows: gross social product X, national income produced Y, fixed capital stock K, fixed capital investment I, wages W, employment L, population N, labor productivity (national income-to-employment ratio) l, capital stock-to-output ratio k, material expenditures-to-output ratio m.

Gross Social Product. According to Soviet methodology, this indicator is defined as the sum of gross values of output by sector of the economy. As one can see in Table 2, the republics with the highest share in total GSP - the RSFSR and the Ukraine - are among those having the lowest rates of economic growth, with totals of 18 and 16 percent respectively. The Asian republics are among those with highest rates of growth: Kazakhstan and Kirgizia, along with Belorussia, will increase the value of their GSP by 29 percent in 1981-1985, while Turkmenistan will increase its by 37 percent.

National Income. National income of republics is determined as the sum of net values of output by sector of the economy. The target for the growth of national income in the 11th Five-Year Plan equals 18 percent, and so does the result suggested by our model. The fact that the model estimates the same rate of growth for gross social product and national income leads to the conclusion that the target to reduce the share of material expenditures (intermediate product) in the total output, will not

Table 2. Growth of the Main Macroindicators by Republic in 1981-1985

| | Gross Social Product X | National Income Y | Capital Stock K | Investment I | Labor L | Wages W | Population N | Productivity l | Capital-to-Output Ratio k | Material Expenditures-to-Output Ratio m |
|---|---|---|---|---|---|---|---|---|---|---|
| U.S.S.R. | 18 | 18 | 21 | 18 | 8 | 17 | 5 | 10 | +2 | 0 |
| Armenia | 20 | 20 | 19 | 14 | 13 | 17 | 10 | 6 | -1 | 0 |
| Azerbaidzhan | 23 | 24 | 17 | 17 | 14 | 20 | 11 | 8 | -5 | -1 |
| Belorussia | 29 | 30 | 23 | 20 | 9 | 27 | 4 | 18 | -4 | -1 |
| Estonia | 17 | 19 | 22 | 17 | 3 | 16 | 1 | 16 | +4 | -1 |
| Georgia | 32 | 29 | 26 | 18 | 12 | 22 | 6 | 14 | -4 | +2 |
| Kazakhstan | 29 | 28 | 27 | 17 | 13 | 29 | 9 | 13 | -1 | +1 |
| Kirgizia | 29 | 27 | 24 | 16 | 15 | 31 | 13 | 10 | -4 | +1 |
| Latvia | 15 | 13 | 22 | 18 | 1 | 10 | 1 | 12 | +6 | +1 |
| Lithuania | 24 | 21 | 32 | 22 | 6 | 20 | 3 | 14 | +8 | +2 |
| Moldavia | 18 | 17 | 26 | 20 | 10 | 16 | 6 | 6 | +6 | +1 |
| R.S.F.S.R. | 18 | 18 | 20 | 19 | 8 | 16 | 3 | 10 | +1 | 0 |
| Tadzhikistan | 22 | 20 | 24 | 15 | 17 | 16 | 16 | 3 | +2 | +1 |
| Turkmenistan | 37 | 32 | 25 | 18 | 17 | 31 | 15 | 12 | -9 | +4 |
| Ukraine | 16 | 15 | 21 | 16 | 5 | 14 | 2 | 9 | +4 | +1 |
| Uzbekistan | 25 | 24 | 28 | 21 | 17 | 23 | 16 | 6 | +5 | +1 |

be met, as it has not been in the past. It should be mentioned here that the ambitious target of reducing consumption of rolled metals by 18-20 percent in the 11th Five-Year plan, along with energy and fuels, and to increase the share of net value added accordingly was set by the party and planning authorities. The authorities, of course, hardly believe this is realistic, but both planning discipline and the mobilizing effect of planning require imposing tough constraints and goals.

Professor Koropeckyj (1981) estimated gross national product per capita by republic in 1970. Estonia, Latvia, and the R.S.F.S.R. gained the first, second, and third places respectively. It is interesting that, according to our calculations, these republics will be leading in per capita production of national income fifteen years hence, i.e., in 1985. The three lagging republics -- Uzbekistan, Azerbaidzhan, and Turkmenistan -- also turn out to be the same in both studies.

Fixed Capital Stock. The transition from extensive to intensive economic growth declared by the Soviets for the 10th and 11th five-year plans has to result in the gradual restructuring of the composition of fixed capital stock. The role of machinery and equipment in the investment process has to grow at the expense of construction industry. This is the result of the investment process changing its emphasis from building new capacities to reconstruction and modernization of existing ones. Nevertheless, new capacities will probably grow at a speed greater than that desired by Soviet authorities, which is an indication that obsolete equipment will still continue to be operated. In the 10th Five-Year Plan about 30 percent of the total product mix of machine-building ministries

had been manufactured for more than ten years. There is no indication that this trend will be reversed and that the machine-building industry will introduce more rapid technological change in the Soviet economy. The results indicated in Table 2 show that the leading republics -- the R.S.F.S.R and the Ukraine -- will lag behind some of the others, with 20 and 21 percent increases in fixed capital stock respectively.

Fixed Capital Investment. According to the estimates in Table 2, the variation of investment growth among republics is rather limited, from 15 percent for Tadzhikistan, the lowest, to 22 percent for Lithuania, the highest. Of course, high or low growth for these republics is not of much importance by itself because it is measured relative to small absolute values, where insignificant fluctuations can result in significant rates of growth. Total, all-union, fixed investment growth equals 18 percent according to our calculations. This is in excess of the 11th Five-Year Plan target of 11 percent growth. Investment is such a volatile indicator that the relevant equations should, of course, incorporate a judgmental component reflecting the estimate of possible changes in the future policies.

In this respect our results could contain an upward bias. On the other hand, it is not clear whether the planned target of 11 percent growth is the estimate of potential capacities of industries manufacturing investment goods. It might follow as well from a desire to reduce capital investment flows sharply to force the ministries and enterprises to seek alternative ways of increasing productivity. If this assumption is true, then the economy has the productive capacity to surpass the planned target

for investment growth. The actual increase of this indicator was 42 percent in the 9th Five-Year Plan (1971-1975) and 29 percent in the 10th Five-Year Plan (1976-1980). Increasing prices of machinery and equipment is another factor that will contribute to investment growth, although not in physical terms. This is not taken into account in the process of five-year planning which creates a downward bias in estimating output in money terms and an upward bias for output in physical terms.

Labor and Population. Population is an exogenous variable of our model. The projections of the FDAD of the U.S. bureau of Census (1979) were employed with these purposes. They foresee the highest growth rates for Uzbekistan (16%), Tadzhikistan (16%), and Turkmenistan (15%) and lowest for Latvia (1%), Estonia (1%), and the Ukraine (2%). The growth of total population is forecasted at the level of 5 percent whereas, according to our calculations, total employment can increase by about 8 percent in the five-year period. The discrepancy indicates that some reserves of manpower still exist in the Soviet economy. However, the question of how to engage these resources in material production is beyond the scope of an econometric study, and additional analysis will be needed. Table 2 shows that the pattern of employment growth distribution reflects that of population, the result that could be expected.

Labor Productivity. The labor productivity indicator is defined in the Soviet economy as average product, i.e., the output-to-labor ratio. The 1979 Resolution of the Central Committee and Council of Ministers brought an essential change in the measurement of this indicator.

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According to the Resolution, net value of output (chistaia produktsiia), not grow value of output (valovaia produktsiia) as before, is to be used for these purposes. The reason for this was to avoid the double counting of material expenditures in the expression of productivity and to isolate the contribution of each enterprise on the basis of net value added per employee. In many cases, however, the initial version is still being employed.

It is easy to show that the create of productivity growth can be expressed as follows:

$$r_{\pi} = (r_u - r_1)/R_1, \tag{12}$$

where $r_{\pi}$ = rate of increase of productivity in current period; $r_u$ = rate of increase of the value of output; $r_1$ = rate of increase of manpower in current period; $R_1$ = growth ratio of manpower, i.e., $R_1 = 1 + r_1$.

Evidently, the higher the rate $r_u$ of increase of output, the higher the productivity growth measured by formula (12). But the point is that the output is expressed in money terms and, therefore, depends on prices. The rate of its increase, if computed discretely at successive periods of time, could be decomposed as follows:

$$r_u = r_x + r_p + r_x r_p, \tag{13}$$

where $r_u$ = rate of increase of output in current period; $r_x$ = rate of

increase of real output or output in physical terms for a monoproduct industry; $r_p$ = rate of price increase measured usually by a price index. When the third term on the right-hand side of formula (13) is negligible, real economic growth is the difference between actual growth and that of prices.

Although only the real-growth component in formula (13) contributes to productivity growth, no distinction between the two components -- output growth in physical terms and that due to the price growth -- is made in formula (12). Therefore, national economic plans account for bogus productivity growth if there are significant price increases. For example, the change of product mix leading to the growth of average prices means productivity growth from the enterprise's standpoint. Not without reason the change of the product mix is one of the most important factors of the productivity growth accounted for at all levels of the economy.

Even with the above reservations, our calculations indicate very modest productivity growth in the 1981-1985 five-year period. With total all-union growth of labor productivity of 10 percent during this period, the republic data vary from 3 percent for Tadzhikistan to 18 percent for Belorussia. The largest republics -- the RSFSR and the Ukraine -- exhibit almost equal productivity growth of 9-10 percent which is equivalent to a 1.8 percent growth rate per annum. These results demonstrate that the plan target calling for 20 percent productivity growth in the 11th Five-Year Plan will not be met.

Capital-to-Output Ratio. This ratio changes insignificantly over time, but much attention is paid to it in planning. It is viewed as an

indication of success in reducing the costs of construction and installation work and newly-introduced equipment. It also characterizes the productivity of machinery and equipment and the degree of waste of capacities. Therefore, according to the mobilizing effect of planning, this ratio should tend to decline. Our calculations produced mixed results. As one can see from Table 2, the capital-to-output ratio will decrease for seven republics and increase for eight of them. Most important is that this ratio will increase for both leading republics -- the R.S.F.S.R. and the Ukraine -- and for the economy as a whole.

Material Expenditures-to-Output Ratio. From our calculations, it follows that this ratio will increase for ten republics, decline for three republics, and will not change for two of them. Despite ever-growing material expenditures per ruble of output, two unprecedent programs -- for saving metals and energy -- were declared in operation for the 11th Five-Year Plan. The results of our calculations and the analysis of other factors indicate that these programs will not be successful. The strict enforcement of the tough constraints on major material inputs imposed by these programs would lead to a further slump in Soviet economic growth. In spite of all the attempts to reverse events, the Soviet economy still depends on extensive factors of economic growth, and substitutes for them are far beyond reach.

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# APPENDIX I:

## List of Equations

1: Y11 == Z11*X11-D11

2: Y12 == Z12*X12-D12

3: Y13 == Z13*X13-D13

4: Y14 == Z14*X14-D14

5: Y15 == Z15*X15-D15

6: P11 == Y11-W11

7: P12 == Y12-W12

8: P13 == Y13-W13

9: P14 == Y14-W14

10: P15 == Y15-W15

11: P1 == P11+P12+P13+P14+P15

12: I11 == DD11+II11

13: I12 == DD12+II12

14: I13 == DD13+II13

15: I14 == DD14+II14

16: I15 == DD15+II15

17: U11 == W11+W12+W13+W14+W15

18: W1 == U11+U12

19: X1 == X11+X12+X13+X14+X15+T1

20: Y1 == Y11+Y12+Y13+Y14+Y15+T1

```
21:       Y21 == Z21*X21-D21
22:       Y22 == Z22*X22-D22
23:       Y23 == Z23*X23-D23
24:       Y24 == Z24*X24-D24
25:       Y25 == Z25*X25-D25
26:       P21 == Y21-W21
27:       P22 == Y22-W22
28:       P23 == Y23-W23
29:       P24 == Y24-W24
30:       P25 == Y25-W25
31:       P2 == P21+P22+P23+P24+P25
32:       I21 == DD21+II21
33:       I22 == DD22+II22
34:       I23 == DD23+II23
35:       I24 == DD24+II24
36:       I25 == DD25+II25
37:       U21 == W21+W22+W23+W24+W25
38:       W2 == U21+U22
39:       X2 == X21+X22+X23+X24+X25+T2
40:       Y2 == Y21+Y22+Y23+Y24+Y25+T2
41:       Y31 == Z31*X31-D31
42:       Y32 == Z32*X32-D32
43:       Y33 == Z33*X33-D33
44:       Y34 == Z34*X34-D34
45:       Y35 == Z35*X35-D35
46:       P31 == Y31-W31
47:       P32 == Y32-W32
48:       P33 == Y33-W33
49:       P34 == Y34-W34
50:       P35 == Y35-W35
51:       P3 == P31+P32+P33+P34+P35
52:       I31 == DD31+II31
53:       I32 == DD32+II32
```

```
54:       I33 == DD33+II33
55:       I34 == DD34+II34
56:       I35 == DD35+II35
57:       U31 == W31+W32+W33+W34+W35
58:       W3 == U31+U32
59:       X3 == X31+X32+X33+X34+X35+T3
60:       Y3 == Y31+Y32+Y33+Y34+Y35+T3
61:       Y41 == Z41*X41-D41
62:       Y42 == Z42*X42-D42
63:       Y43 == Z43*X43-D43
64:       Y44 == Z44*X44-D44
65:       Y45 == Z45*X45-D45
66:       P41 == Y41-W41
67:       P42 == Y42-W42
68:       P43 == Y43-W43
69:       P44 == Y44-W44
70:       P45 == Y45-W45
71:       P4 == P41+P42+P43+P44+P45
72:       I41 == DD41+II41
73:       I42 == DD42+II42
74:       I43 == DD43+II43
75:       I44 == DD44+II44
76:       I45 == DD45+II45
77:       U41 == W41+W42+W43+W44+W45
78:       W4 == U41+U42
79:       X4 == X41+X42+X43+X44+X45+T4
80:       Y4 == Y41+Y42+Y43+Y44+Y45+T4
81:       Y51 == Z51*X51-D51
82:       Y52 == Z52*X52-D52
83:       Y53 == Z53*X53-D53
84:       Y54 == Z54*X54-D54
85:       Y55 == Z55*X55-D55
86:       P51 == Y51-W51
```

```
87:       P52 == Y52-W52
88:       P53 == Y53-W53
89:       P54 == Y54-W54
90:       P55 == Y55-W55
91:       P5 == P51+P52+P53+P54+P55
92:       I51 == DD51+II51
93:       I52 == DD52+II52
94:       I53 == DD53+II53
95:       I54 == DD54+II54
96:       I55 == DD55+II55
97:       U51 == W51+W52+W53+W54+W55
98:       W5 == U51+U52
99:       X5 == X51+X52+X53+X54+X55+T5
100:      Y5 == Y51+Y52+Y53+Y54+Y55+T5
101:      Y61 == Z61*X61-D61
102:      Y62 == Z62*X62-D62
103:      Y63 == Z63*X63-D63
104:      Y64 == Z64*X64-D64
105:      Y65 == Z65*X65-D65
106:      P61 == Y61-W61
107:      P62 == Y62-W62
108:      P63 == Y63-W63
109:      P64 == Y64-W64
110:      P65 == Y65-W65
111:      P6 == P61+P62+P63+P64+P65
112:      I61 == DD61+II61
113:      I62 == DD62+II62
114:      I63 == DD63+II63
115:      I64 == DD64+II64
116:      I65 == DD65+II65
117:      U61 == W61+W62+W63+W64+W65
118:      W6 == U61+U62
119:      X6 == X61+X62+X63+X64+X65+T6
```

```
120:        Y6 == Y61+Y62+Y63+Y64+Y65+T6
121:        Y71 == Z71*X71-D71
122:        Y72 == Z72*X72-D72
123:        Y73 == X73*X73-D73
124:        Y74 == Z74*X74-D74
125:        Y75 == Z75*X75-D75
126:        P71 == Y71-W71
127:        P72 == Y72-W72
128:        P73 == Y73-W73
129:        P74 == Y74-D74
130:        P75 == Y75-D75
131:        P7 == P71+P72+P73+P74+P75
132:        I71 == DD71+II71
133:        I72 == DD72+II72
134:        I73 == DD73+II73
135:        I74 == DD74+II74
136:        I75 == DD75+II75
137:        U71 == W71+W72+W73+W74+W75
138:        W7 == Y71+U72
139:        X7 == X71+X72+X73+X74+X75+T7
140:        Y7 == Y71+Y72+Y73+Y74+Y75+T7
141:        Y81 == Z81*X81-D81
142:        Y82 == Z82*X82-D82
143:        Y83 == Z83*X83-D83
144:        Y84 == Z84*X84-D84
145:        Y85 == Z85*X85-D85
146:        P81 == Y81-W81
147:        P82 == Y82-W82
148:        P83 == Y83-W83
149:        P84 == Y84-W84
150:        P85 == Y85-W85
151:        P8 == P81+P82+P83+P84+P85
152:        I81 == DD81+II81
```

```
153:        I82 == DD82+II82
154:        I83 == DD83+II83
155:        I84 == DD84+II84
156:        I85 == DD85+II85
157:        U81 == W81+W82+W83+W84+W85
158:        W8 == U81+U82
159:        X8 == X81+X82+X83+X84+X85+T8
160:        Y8 == Y81+Y82+Y83+Y84+Y85+T8
161:        Y91 == Z91*X91-D91
162:        Y92 == Z92*X92-D92
163:        Y93 == Z93*X93-D93
164:        Y94 == Z94*X94-D94
165:        Y95 == Z95*X95-D95
166:        P91 == Y91-W91
167:        P92 == Y92-W92
168:        P93 == Y93-W93
169:        P94 == Y94-W94
170:        P95 == Y95-W95
171:        P9 == P91+P92+P93+P94+P85
172:        I91 == DD91+II91
173:        I92 == DD92+II92
174:        I93 == DD93+II93
175:        I94 == DD94+II94
176:        I95 == DD95+II95
177:        U91 == W91+W92+W93+W94+W95
178:        W9 == U91+U92
179:        X9 == X91+X92+X93+X94+X95+T9
180:        Y9 == Y91+Y92+Y93+Y94+Y95+T9
181:        Y101 == Z101*X101-D101
182:        Y102 == Z102*X102-D102
183:        Y103 == Z103*X103-D103
184:        Y104 == Z104*X104-D104
185:        Y105 == Z105*X105-D105
```

```
186:        P101 == Y101-W101
187:        P102 == Y102-W102
188:        P103 == Y103-W103
189:        P104 == Y104-W104
190:        P105 == Y105-W105
191:        P10 == P101+P102+P103+P104+P105
192:        I101 == DD101+II101
193:        I102 == DD102+II102
194:        I103 == DD103+II103
195:        I104 == DD104+II104
196:        I105 == DD105+II105
197:        U101 == W101+W102+W103+W104+W105
198:        W10 == U101+U102
199:        X10 == X101+X102+X103+X104+X105+T10
200:        Y10 == Y101+Y102+Y103+Y104+Y105+T10
201:        Y111 == Z111*X111-D111
202:        Y112 == Z112*X112-D112
203:        Y113 == Z113*X113-D113
204:        Y114 == Z114*X114-D114
205:        Y115 == Z115*X115-D115
206:        P111 == Y111-W111
207:        P112 == Y112-W112
208:        P113 == Y113-W113
209:        P114 == Y114-W114
210:        P115 == Y115-W115
211:        PP11 == P111+P112+P113+P114+P115
212:        I111 == DD111+II111
213:        I112 == DD112+II112
214:        I113 == DD113+II113
215:        I114 == DD114+II114
216:        I115 == DD115+II115
217:        U111 == W111+W112+W113+W114+W115
218:        WW11 == U111+U112
```

```
219:        XX11 == X111+X112+X113+X114+X115+T11
220:        YY11 == Y111+Y112+Y113+Y114+Y115+T11
221:        Y121 == Z121*X121-D121
222:        Y122 == Z122*X122-D122
223:        Y123 == Z123*X123-D123
224:        Y124 == Z124*X124-D124
225:        Y125 == Z125*X125-D125
226:        P121 == Y121-W121
227:        P122 == Y122-W122
228:        P123 == Y123-W123
229:        P124 == Y124-W124
230:        P125 == Y125-W125
231:        PP12 == P121+P122+P123+P124+P125
232:        I121 == DD121+II121
233:        I122 == DD122+II122
234:        I123 == DD123+II123
235:        I124 == DD124+II124
236:        I125 == DD125+II125
237:        U121 == W121+W122+W123+W124+W125
238:        WW12 == U121+U122
239:        XX12 == X121+X122+X123+X124+X125+T12
240:        YY12 == Y121+Y122+Y123+Y124+Y125+T12
241:        Y131 == Z131*X131-D131
242:        Y132 == Z132*X132-D132
243:        Y133 == Z133*X133-D133
244:        Y134 == Z134*X134-D134
245:        Y135 == Z135*X135-D135
246:        P131 == Y131-W131
247:        P132 == Y132-W132
248:        P133 == Y133-W133
249:        P134 == Y134-W134
250:        P135 == Y135-W135
251:        PP13 == P131+P132+P133+P134+P135
```

```
252:        I131 == DD131+II131
253:        I132 == DD132+II132
254:        I133 == DD133+II133
255:        I134 == DD134+II134
256:        I135 == DD135+II135
257:        U131 == W131+W132+W133+W134+W135
258:        WW13 == U131+U132
259:        XX13 == X131+X132+X133+X134+X135+T13
260:        YY13 == Y131+Y132+Y133+Y134+Y135+T13
261:        Y141 == Z141*X141-D141
262:        Y142 == Z142*X142-D142
263:        Y143 == Z143*X143-D143
264:        Y144 == Z144*X144-D144
265:        Y145 == Z145*X145-D145
266:        P141 == Y141-W141
267:        P142 == Y142-W142
268:        P143 == Y143-W143
269:        P144 == Y144-W144
270:        P145 == Y145-W145
271:        PP14 == P141+P142+P143+P144+P145
272:        I141 == DD141+II141
273:        I142 == DD142+II142
274:        I143 == DD143+II143
275:        I144 == DD144+II144
276:        I145 == DD145+II145
277:        U141 == W141+W142+W143+W144+W145
278:        WW14 == U141+U142
279:        XX14 == X141+X142+X143+X144+X145+T14
280:        YY14 == Y141+Y142+Y143+Y144+Y145+T14
281:        Y151 == Z151*X151-D151
282:        Y152 == Z152*X152-D152
283:        Y153 == Z153*X153-D153
284:        Y154 == Z154*X154-D154
```

```
285:        Y155 == Z155*X155-D155
286:        P151 == Y151-W151
287:        P152 == Y152-W152
288:        P153 == Y153-W153
289:        P154 == Y154-W154
290:        P155 == Y155-W155
291:        PP15 == P151+P152+P153+P154+P155
292:        I151 == DD151+II151
293:        I152 == DD152+II152
294:        I153 == DD153+II153
295:        I154 == DD154+II154
296:        I155 == DD155+II155
297:        U151 == W151+W152+W153+W154+W155
298:        WW15 == U151+U152
299:        XX15 == X151+X152+X153+X154+X155+T15
300:        YY15 == Y151+Y152+Y153+Y154+Y155+T15
301:        R1 == T1+T2+T3+T4+T5+T6+T7+T8+T9+T10+T11+T12+T13+T14+T15
302:        R2 = K1+K2*(P1+P2+P3+P4+P5+P6+P7+P8+P9+P10+PP11+PP12+PP13+PP14+PP15)
303:        R3 = L1+L2*(W1+W2+W3+W4+W5+W6+W7+W8+W9+W10+WW11+WW12+WW13+WW14+WW15)
304:        R4 = M1+M2*(W1+W2+W3+W4+W5+W6+W7+W8+W9+W10+WW11+WW12+WW13+WW14+WW15)
305:        R5 = NB1+NB2*(R1+R2+R3+R4)
306:        R == R1+R2+R3+R4+R5
307:        E1 = O1+O2*R
308:        E2 = PB1+PB2*R
309:        E3 = Q1+Q2*R
310:        B = O11+O22*E1
311:        X11 = A111+A112*LOG(K11)+A113*LOG(L11)
312:        X12 = A121+A122*LOG(K12)+A123*LOG(L12)+A124*LOG(Q12)
313:        X13 = A131+A132*LOG(K13)+A133*LOG(L13)
314:        X14 = A141+A142*LOG(K14)+A143*LOG(L14)
315:        X15 = A151+A152*LOG(K15)+A153*LOG(L15)
316:        K11 = K11(-1)+B111+B112*I11+B113*I11(-1)+B114*I11(-2)+B115*EX
317:        K12 = K12(-1)+B121+B122*I12+B123*I12(-1)+B124*I12(-2)+B125*EX
```

```
318:        K13 = K13(-1)+B131+B132*I13+B133*I13(-1)+B134*I13(-2)+B135*EX
319:        K14 = K14(-1)+B141+B142*I14+B143*I14(-1)+B144*I14(-2)+B145*EX
320:        K15 = K15(-1)+B151+B152*I15+B153*I15(-1)+B154*I15(-2)+B155*EX
321:        DD11 = C111+C112*D11
322:        DD12 = C121+C122*D12
323:        DD13 = C131+C132*D13
324:        DD14 = C141+C142*D14
325:        DD15 = C151+C152*D15
326:        II11 = CC111+CC112*B1
327:        II12 = CC121+CC122*B1
328:        II13 = CC131+CC132*B1
329:        II14 = CC141+CC142*B1
330:        II15 = CC151+CC152*B1
331:        D11 = AA111+AA112*K11
332:        D12 = AA121+AA122*K12
333:        D13 = AA131+AA132*K13
334:        D14 = AA141+AA142*K14
335:        D15 = AA151+AA152*K15
336:        L11 = E111+E112*N1
337:        L12 = E121+E122*N1
338:        L13 = E131+E132*N1
339:        L14 = E141+E142*N1
340:        L15 = E151+E152*N1
341:        W11 = F111+F112*Y11
342:        W12 = F121+F122*Y12
343:        W13 = F131+F132*Y13
344:        W14 = F141+F142*Y14
345:        W15 = F151+F152*Y15
346:        T1 = G11+G12*(X11+X12+X13+X14+X15)
347:        U12 = H11+H12*(W11+W12+W13+W14+W15)
348:        B1 = J11+J12*B
349:        Q12 = V121+V122*T
350:        K21 = K21(-1)+B211+B212*I21+B213*I21(-1)+B214*I21(-2)+B215*EX
```

```
K22 = K22(-1)+B221+B222*I22+B223*I22(-1)+B224*I22(-2)+B225*EX
K23 = K23(-1)+B231+B232*I23+B233*I23(-1)+B234*I23(-2)+B235*EX
K24 = K24(-1)+B241+B242*I24+B243*I24(-1)+B244*I24(-2)+B245*EX
K25 = K25(-1)+B251+B252*I25+B253*I25(-1)+B254*I25(-2)+B255*EX
DD21 = C211+C212*D21
DD22 = C221+C222*D22
DD23 = C231+C232*D23
DD24 = C241+C242*D24
DD25 = C251+C252*D25
II21 = CC211+CC212*B2
II22 = CC221+CC222*B2
II23 = CC231+CC232*B2
II24 = CC241+CC242*B2
II25 = CC251+CC252*B2
X21 = A211+A212*LOG(K21)+A213*LOG(L21)
X22 = A221+A222*LOG(K22)+A223*LOG(L22)+A224*LOG(Q22)
X23 = A231+A232*LOG(K23)+A233*LOG(L23)
X24 = A241+A242*LOG(K24)+A243*LOG(L24)
X25 = A251+A252*LOG(K25)+A253*LOG(L25)
D21 = AA211+AA212*K21
D22 = AA221+AA222*K22
D23 = AA231+AA232*K23
D24 = AA241+AA242*K24
D25 = AA251+AA252*K25
L21 = E211+E212*N2
L22 = E221+E222*N2
L23 = E231+E232*N2
L24 = E241+E242*N2
L25 = E251+E252*N2
W21 = F211+F212*Y21
W22 = F221+F222*Y22
W23 = F231+F232*Y23
W24 = F241+F242*Y24
```

```
384:        W25 = F251+F252*Y25
385:        T2 = G21+G22*(X21+X22+X23+X24+X25)
386:        U22 = H21+H22*(W21+W22+W23+W24+W25)
387:        B2 = J21+J22*B
388:        Q22 = V221+V222*T
389:        K31 = K31(-1)+B311+B312*I31+B313*I31(-1)+B314*I31(-2)+B315*EX
390:        K32 = K32(-1)+B321+B322*I32+B323*I32(-1)+B324*I32(-2)+B325*EX
391:        K33 = K33(-1)+B331+B332*I33+B333*I33(-1)+B334*I33(-2)+B335*EX
392:        K34 = K34(-1)+B341+B342*I34+B343*I34(-1)+B344*I34(-2)+B345*EX
393:        K35 = K35(-1)+B351+B352*I35+B353*I35(-1)+B354*I35(-2)+B355*EX
394:        DD31 = C311+C312*D31
395:        DD32 = C321+C322*D32
396:        DD33 = C331+C332*D33
397:        DD34 = C341+C342*D34
398:        DD35 = C351+C352*D35
399:        II31 = CC311+CC312*B3
400:        II32 = CC321+CC322*B3
401:        II33 = CC331+CC332*B3
402:        II34 = CC341+CC342*B3
403:        II35 = CC351+CC352*B3
404:        X31 = A311+A312*LOG(K31)+A313*LOG(L31)
405:        X32 = A321+A322*LOG(K32)+A323*LOG(L32)+A324*LOG(Q32)
406:        X33 = A331+A332*LOG(K33)+A333*LOG(L33)
407:        X34 = A341+A342*LOG(K34)+A343*LOG(L34)
408:        X35 = A351+A352*LOG(K35)+A353*LOG(L35)
409:        D31 = AA311+AA312*K31
410:        D32 = AA321+AA322*K32
411:        D33 = AA331+AA332*K33
412:        D34 = AA341+AA342*K34
413:        D35 = AA351+AA352*K35
414:        L31 = E311+E312*N3
415:        L32 = E321+E322*N3
416:        L33 = E331+E332*N3
```

```
417:        L34 = E341+E342*N3
418:        L35 = E351+E352*N3
419:        W31 = F311+F312*Y31
420:        W32 = F321+F322*Y32
421:        W33 = F331+F332*Y33
422:        W34 = F341+F342*Y34
423:        W35 = F351+F352*Y35
424:        T3 = G31+G32*(X31+X32+X33+X34+X35)
425:        U32 = H31+H32*(W31+W32+W33+W34+W35)
426:        B3 = J31+J32*B
427:        Q32 = V321+V322*T
428:        K41 = K41(-1)+B411+B412*I41+B413*I41(-1)+B414*I14(-2)+B415*EX
429:        K42 = K42(-1)+B421+B422*I42+B423*I42(-1)+B424*I42(-2)+B425*EX
430:        K43 = K43(-1)+B431+B432*I43+B433*I43(-1)+B434*I43(-2)+B435*EX
431:        K44 = K44(-1)+B441+B442*I44+B443*I44(-1)+B444*I44(-2)+B445*EX
432:        K45 = K45(-1)+B451+B452*I45+B453*I45(-1)+B454*I45(-2)+B455*EX
433:        DD41 = C411+C412*D41
434:        DD42 = C421+C422*D42
435:        DD43 = C431+C432*D43
436:        DD44 = C441+C442*D44
437:        DD45 = C451+C452*D45
438:        II41 = CC411+CC412*B4
439:        II42 = CC421+CC422*B4
440:        II43 = CC431+CC432*B4
441:        II44 = CC441+CC442*B4
442:        II45 = CC451+CC452*B4
443:        X41 = A411+A412*LOG(K41)+A413*LOG(L41)
444:        X42 = A421+A422*LOG(K42)+A423*LOG(L42)+A424*LOG(Q42)
445:        X43 = A431+A432*LOG(K43)+A433*LOG(L43)
446:        X44 = A441+A442*LOG(K44)+A443*LOG(L44)
447:        X45 = A451+A452*LOG(K45)+A453*LOG(L45)
448:        D41 = AA411+AA412*K41
449:        D42 = AA421+AA422*K42
```

```
450:        D43 = AA431+AA432*K43
451:        D45 = AA451+AA452*K45
452:        D44 = AA441+AA442*K44
453:        L41 = E411+E412*N4
454:        L42 = E421+E422*N4
455:        L43 = E431+E432*N4
456:        L44 = E441+E442*N4
457:        L45 = E451+E452*N4
458:        W41 = F411+F412*Y41
459:        W42 = F421+F422*Y42
460:        W43 = F431+F432*Y43
461:        W44 = F441+F442*Y44
462:        W45 = F451+F452*Y45
463:        T4 = G41+G42*(X41+X42+X43+X44+X45)
464:        U42 = H41+H42*(W41+W42+W43+W44+W45)
465:        B4 = J41+J42*B
466:        Q42 = V421+V422*T
467:        K51 = K51(-1)+B511+B512*I51+B513*I51(-1)+B514*I51(-2)+B515*EX
468:        K52 = K52(-1)+B521+B522*I52+B523*I52(-1)+B524*I52(-2)+B525*EX
469:        K53 = K53(-1)+B531+B532*I53+B533*I53(-1)+B534*I53(-2)+B535*EX
470:        K54 = K54(-1)+B541+B542*I54+B543*I54(-1)+B544*I54(-2)+B545*EX
471:        K55 = K55(-1)+B551+B552*I55+B553*I55(-1)+B554*I55(-2)+B555*EX
472:        DD51 = C511+C512*D51
473:        DD52 = C521+C522*D52
474:        DD53 = C531+C532*D53
475:        DD54 = C541+C542*D54
476:        DD55 = C551+C552*D55
477:        II51 = CC511+CC512*B5
478:        II52 = CC521+CC522*B5
479:        II53 = CC531+CC532*B5
480:        II54 = CC541+CC542*B5
481:        II55 = CC551+CC552*B5
482:        X51 = A511+A512*LOG(K51)+A513*LOG(L51)
```

```
483:        X52 = A521+A522*LOG(K52)+A523*LOG(L52)+A524*LOG(Q52)
484:        X53 = A531+A532*LOG(K53)+A533*LOG(L53)
485:        X54 = A541+A542*LOG(K54)+A543*LOG(L54)
486:        X55 = A551+A552*LOG(K55)+A553*LOG(L55)
487:        D51 = AA511+AA512*K51
488:        D52 = AA521+AA522*K52
489:        D53 = AA531+AA532*K53
490:        D54 = AA541+AA542*K54
491:        D55 = AA551+AA552*K55
492:        L51 = E511+E512*N5
493:        L52 = E521+E522*N5
494:        L53 = E531+E532*N5
495:        L54 = E541+E542*N5
496:        L55 = E551+E552*N5
497:        W51 = F511+F512*Y51
498:        W52 = F521+F522*Y52
499:        W53 = F531+F532*Y53
500:        W54 = F541+F542*Y54
501:        W55 = F551+F552*Y55
502:        T5 = G51+G52*(X51+X52+X53+X54+X55)
503:        U52 = H51+H52*(W51+W52+W53+W54+W55)
504:        B5 = J51+J52*B
505:        Q52 = V521+V522*T
506:        K61 = K61(-1)+B611+B612*I61+B613*I61(-1)+B614*I61(-2)+B615*EX
507:        K62 = K62(-1)+B621+B622*I62+B623*I62(-1)+B624*I62(-2)+B625*EX
508:        K63 = K63(-1)+B631+B632*I63+B633*I63(-1)+B634*I63(-2)+B635*EX
509:        K64 = K64(-1)+B641+B642*I64+B643*I64(-1)+B644*I64(-2)+B645*EX
510:        K65 = K65(-1)+B651+B652*I65+B653*I65(-1)+B654*I65(-2)+B655*EX
511:        DD61 = C611+C612*D61
512:        DD62 = C621+C622*D62
513:        DD63 = C631+C632*D63
514:        DD64 = C641+C642*D64
515:        DD65 = C651+C652*D65
```

```
516:        II61 = CC611+CC612*B6
517:        II62 = CC621+CC622*B6
518:        II63 = CC631+CC632*B6
519:        II64 = CC641+CC642*B6
520:        II65 = CC651+CC652*B6
521:        X61 = A611+A612*LOG(K61)+A613*LOG(L61)
522:        X62 = A621+A622*LOG(K62)+A623*LOG(L62)+A624*LOG(Q62)
523:        X63 = A631+A632*LOG(K63)+A633*LOG(L63)
524:        X64 = A641+A642*LOG(K64)+A643*LOG(L64)
525:        X65 = A651+A652*LOG(K65)+A653*LOG(L65)
526:        D61 = AA611+AA612*K61
527:        D62 = AA621+AA622*K62
528:        D63 = AA631+AA632*K63
529:        D64 = AA641+AA642*K64
530:        D65 = AA651+AA652*K65
531:        L61 = E611+E612*N6
532:        L62 = E621+E622*N6
533:        L63 = E631+E632*N6
534:        L64 = E641+E642*N6
535:        L65 = E651+E652*N6
536:        W61 = F611+F612*Y61
537:        W62 = F621+F622*Y62
538:        W63 = F631+F632*Y63
539:        W64 = F641+F642*Y64
540:        W65 = F651+F652*Y65
541:        T6 = G61+G62*(X61+X62+X63+X64+X65)
542:        U62 = H61+H62*(W61+W62+W63+W64+W65)
543:        B6 = J61+J62*B
544:        Q62 = V621+V622*T
545:        K71 = K71(-1)+B711+B712*I71+B713*I71(-1)+B714*I71(-2)+B715*EX
546:        K72 = K72(-1)+B721+B722*I72+B723*I72(-1)+B724*I72(-2)+B725*EX
547:        K73 = K73(-1)+B731+B732*I73+B733*I73(-1)+B734*I73(-2)+B735*EX
548:        K74 = K74(-1)+B741+B742*I74+B743*I74(-1)+B744*I74(-2)+B745*EX
```

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549:      K75 = K75(-1)+B751+B752*I75+B753*I75(-1)+B754*I75(-2)+B755*EX
550:      DD71 = C711+C712*D71
551:      DD72 = C721+C722*D72
552:      DD73 = C731+C732*D73
553:      DD74 = C741+C742*D74
554:      DD75 = C751+C752*D75
555:      II71 = CC711+CC712*B7
556:      II72 = CC721+CC722*B7
557:      II73 = CC731+CC732*B7
558:      II74 = CC741+CC742*B7
559:      II75 = CC751+CC752*B7
560:      X71 = A711+A712*LOG(K71)+A713*LOG(L71)
561:      X72 = A721+A722*LOG(K72)+A723*LOG(L72)+A724*LOG(Q72)
562:      X73 = A731+A732*LOG(K73)+A733*LOG(L73)
563:      X74 = A741+A742*LOG(K74)+A743*LOG(L74)
564:      X75 = A751+A752*LOG(K75)+A753*LOG(L75)
565:      D71 = AA711+AA712*K71
566:      D72 = AA721+AA722*K72
567:      D73 = AA731+AA732*K73
568:      D74 = AA741+AA742*K74
569:      D75 = AA751+AA752*K75
570:      L71 = E711+E712*N7
571:      L72 = E721+E722*N7
572:      L73 = E731+E732*N7
573:      L74 = E741+E742*N7
574:      L75 = E751+E752*N7
575:      W71 = F711+F712*Y71
576:      W72 = F721+F722*Y72
577:      W73 = F731+F732*Y73
578:      W74 = F741+F742*Y74
579:      W75 = F751+F752*Y75
580:      T7 = G71+G72*(X71+X72+X73+X74+X75)
581:      U72 = H71+H72*(W71+W72+W73+W74+W75)
```

```
582:        B7 = J71+J72*B
583:        Q72 = V721+V722*T
584:        K81 = K81(-1)+B811+B812*I81+B813*I81(-1)+B814*I81(-2)+B815*EX
585:        K82 = K82(-1)+B821+B822*I82+B823*I82(-1)+B824*I82(-2)+B825*EX
586:        K83 = K83(-1)+B831+B832*I83+B833*I83(-1)+B834*I83(-2)+B835*EX
587:        K84 = K84(-1)+B841+B842*I84+B843*I84(-1)+B844*I84(-2)+B845*EX
588:        K85 = K85(-1)+B851+B852*I85+B853*I85(-1)+B854*I85(-2)+B855*EX
589:        DD81 = C811+C812*D81
590:        DD82 = C821+C822*D82
591:        DD83 = C831+C832*D83
592:        DD84 = C841+C842*D84
593:        DD85 = C851+C852*D85
594:        II81 = CC811+CC812*B8
595:        II82 = CC821+CC822*B8
596:        II83 = CC831+CC832*B8
597:        II84 = CC841+CC842*B8
598:        II85 = CC851+CC852*B8
599:        X81 = A811+A812*LOG(K81)+A813*LOG(L81)
600:        X82 = A821+A822*LOG(K82)+A823*LOG(L82)+A824*LOG(Q82)
601:        X83 = A831+A832*LOG(K83)+A833*LOG(L83)
602:        X84 = A841+A842*LOG(K84)+A843*LOG(L84)
603:        X85 = A851+A852*LOG(K85)+A853*LOG(L85)
604:        D81 = AA811+AA812*K81
605:        D82 = AA821+AA822*K82
606:        D83 = AA831+AA832*K83
607:        D84 = AA841+AA842*K84
608:        D85 = AA851+AA852*K85
609:        L81 = E811+E812*N8
610:        L82 = E821+E822*N8
611:        L84 = E841+E842*N8
612:        L83 = E831+E832*N8
613:        L85 = E851+E852*N8
614:        W81 = F811+F812*Y81
```

```
615:        W82 = F821+F822*Y82
616:        W83 = F831+F832*Y83
617:        W84 = F841+F842*Y84
618:        W85 = F851+F852*Y85
619:        T8 = G81+G82*(X81+X82+X83+X84+X85)
620:        U82 = H81+H82*(W81+W82+W83+W84+W85)
621:        B8 = J81+J82*B
622:        Q82 = V821+V822*T
623:        K91 = K91(-1)+B911+B912*I91+B913*I91(-1)+B914*I91(-2)+B915*EX
624:        K92 = K92(-1)+B921+B922*I92+B923*I92(-1)+B924*I92(-2)+B925*EX
625:        K93 = K93(-1)+B931+B932*I93+B933*I93(-1)+B934*I93(-2)+B935*EX
626:        K94 = K94(-1)+B941+B942*I94+B943*I94(-1)+B944*I94(-2)+B945*EX
627:        K95 = K95(-1)+B951+B952*I95+B953*I95(-1)+B954*I95(-2)+B955*EX
628:        DD91 = C911+C912*D91
629:        DD92 = C921+C922*D92
630:        DD93 = C931+C932*D93
631:        DD94 = C941+C942*D94
632:        DD95 = C951+C952*D95
633:        II91 = CC911+CC912*B9
634:        II92 = CC921+CC922*B9
635:        II93 = CC931+CC932*B9
636:        II94 = CC941+CC942*B9
637:        II95 = CC951+CC952*B9
638:        X91 = A911+A912*LOG(K91)+A913*LOG(L91)
639:        X92 = A921+A922*LOG(K92)+A923*LOG(L92)+A924*LOG(Q92)
640:        X93 = A931+A932*LOG(K93)+A933*LOG(L93)
641:        X94 = A941+A942*LOG(K94)+A943*LOG(L94)
642:        X95 = A951+A952*LOG(K95)+A953*LOG(L95)
643:        D91 = AA911+AA912*K91
644:        D92 = AA921+AA922*K92
645:        D93 = AA931+AA932*K93
646:        D94 = AA941+AA942*K94
647:        D95 = AA951+AA952*K95
```

```
648:      L91 = E911+E912*N9
649:      L92 = E921+E922*N9
650:      L93 = E931+E932*N9
651:      L94 = E941+E942*N9
652:      L95 = E951+E952*N9
653:      W91 = F911+F912*Y91
654:      W92 = F921+F922*Y92
655:      W93 = F931+F932*Y93
656:      W94 = F941+F942*Y94
657:      W95 = F951+F952*Y95
658:      T9 = G91+G92*(X91+X92+X93+X94+X95)
659:      U92 = H91+H92*(W91+W92+W93+W94+W95)
660:      B9 = J91+J92*B
661:      Q92 = V921+V922*T
662:      K101 = K101(-1)+B1011+B1012*I101+B1013*I101(-1)+B1014*I101(-2)+B1015*EX
663:      K102 = K102(-1)+B1021+B1022*I102+B1023*I102(-1)+B1024*I102(-2)+B1025*EX
664:      K103 = K103(-1)+B1031+B1032*I103+B1033*I103(-1)+B1034*I103(-2)+B1035*EX
665:      K104 = K104(-1)+B1041+B1042*I104+B1043*I104(-1)+B1044*I104(-2)+B1045*EX
666:      K105 = K105(-1)+B1051+B1052*I105+B1053*I105(-1)+B1054*I105(-2)+B1055*EX
667:      DD101 = C1011+C1012*D101
668:      DD102 = C1021+C1022*D102
669:      DD103 = C1031+C1032*D103
670:      DD104 = C1041+C1042*D104
671:      DD105 = C1051+C1052*D105
672:      II101 = CC1011+CC1012*B10
673:      II102 = CC1021+CC1022*B10
674:      II103 = CC1031+CC1032*B10
675:      II104 = CC1041+CC1042*B10
676:      II105 = CC1051+CC1052*B10
677:      X101 = A1011+A1012*LOG(K101)+A1013*LOG(L101)
678:      X102 = A1021+A1022*LOG(K102)+A1023*LOG(L102)+A1024*LOG(Q102)
679:      X103 = A1031+A1032*LOG(K103)+A1033*LOG(L103)
680:      X104 = A1041+A1042*LOG(K104)+A1043*LOG(L104)
```

```
681:      X105 = A1051+A1052*LOG(K105)+A1053*LOG(L105)
682:      D101 = AA1011+AA1012*K101
683:      D102 = AA1021+AA1022*K102
684:      D103 = AA1031+AA1032*K103
685:      D104 = AA1041+AA1042*K104
686:      D105 = AA1051+AA1052*K105
687:      L101 = E1011+E1012*N10
688:      L102 = E1021+E1022*N10
689:      L103 = E1031+E1032*N10
690:      L104 = E1041+E1042*N10
691:      L105 = E1051+E1052*N10
692:      W101 = F1011+F1012*Y101
693:      W102 = F1021+F1022*Y102
694:      W103 = F1031+F1032*Y103
695:      W104 = F1041+F1042*Y104
696:      W105 = F1051+F1052*Y105
697:      T10 = G101+G102*(X101+X102+X103+X104+X105)
698:      U102 = H101+H102*(W101+W102+W103+W104+W105)
699:      B10 = J101+J102*B
700:      Q102 = V102(?)V1022*T
701:      K111 = K111(-1)+B1111+B1112*I111+B1113*I111(-1)+B1114*I111(-2)+B1115*EX
702:      K112 = K11(?-1)+B1121+B1122*I112+B1123*I112(-1)+B1124*I112(-2)+B1125*EX
703:      K113 = K113(-1)+B1131+B1132*I113+B1133*I113(-1)+B1134*I113(-2)+B1135*EX
704:      K114 = K114(-1)+B1141+B1142*I114+B1143*I114(-1)+B1144*I114(-2)+B1145*EX
705:      K115 = K115(-1)+B1151+B1152*I115+B1153*I115(-1)+B1154*I115(-2)+B1155*EX
706:      DD111 = C1111+C1112*D111
707:      DD112 = C1121+C1122*D112
708:      DD113 = C1131+C1132*D113
709:      DD114 = C1141+C1142*D114
710:      DD115 = C1151+C1152*D115
711:      II111 = CC1111+CC1112*B11
712:      II112 = CC1121+CC1122*B11
713:      II113 = CC1131+CC1132*B11
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714:      II114 = CC1141+CC1142*B11
715:      II115 = CC1151+CC1152*B11
716:      X111 = A1111+A1112*LOG(K111)+A1113*LOG(L111)
717:      X112 = A1121+A1122*LOG(K112)+A1123*LOG(L112)+A1124*LOG(Q112)
718:      X113 = A1131+A1132*LOG(K113)+A1133*LOG(L113)
719:      X114 = A1141+A1142*LOG(K114)+A1143*LOG(L114)
720:      X115 = A1151+A1152*LOG(K115)+A1153*LOG(L115)
721:      D111 = AA1111+AA1112*K111
722:      D112 = AA1121+AA1122*K112
723:      D113 = AA1131+AA1132*K113
724:      D114 = AA1141+AA1142*K114
725:      D115 = AA1151+AA1152*K115
726:      L111 = E1111+E1112*N11
727:      L112 = E1121+E1122*N11
728:      L113 = E1131+E1132*N11
729:      L114 = E1140+E1142*N11
730:      L115 = E1151+E1152*N11
731:      W111 = F1111+F1112*Y111
732:      W112 = F1121+F1122*Y112
733:      W113 = F1131+F1132*Y113
734:      W114 = F1141+F1142*Y114
735:      W115 = F1151+F1152*Y115
736:      T11 = G111+G112*(X111+X112+X113+X114+X115)
737:      U112 = H111+H112*(W111+W112+W113+W114+W115)
738:      B11 = J111+J112*B
739:      Q112 = V1121+V1122*T
740:      K121 = K121(-1)+B1211+B1212*I121+B1213*I121(-1)+B1214*I121(-2)+B1215*EX
741:      K122 = K122(-1)+B1221+B1222*I122+B1223*I122(-1)+B1224*I122(-2)+B1225*EX
742:      K123 = K123(-1)+B1231+B1232*I123+B1233*I123(-1)+B1234*I123(-2)+B1235*EX
743:      K124 = K124(-1)+B1241+B1242*I124+B1243*I124(-1)+B1244*I124(-2)+B1245*EX
744:      K125 = K125(-1)+B1251+B1252*I125+B1253*I125(-1)+B1254*I125(-2)+B1255*EX
745:      DD121 = C1211+C1212*D121
746:      DD122 = C1221+C1222*D122
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747:      DD123 = C1231+C1232*D123
748:      DD124 = C1241+C1242*D124
749:      DD125 = C1251+C1252*D125
750:      II121 = CC1211+CC1212*B12
751:      II122 = CC1221+CC1222*B12
752:      II123 = CC1231+CC1232*B12
753:      II124 = CC1241+CC1242*B12
754:      II125 = CC1251+CC1252*B12
755:      X121 = A1211+A1212*LOG(K121)+A1213*LOG(L121)
756:      X122 = A1221+A1222*LOG(K122)+A1223*LOG(L122)+A1224*LOG(Q122)
757:      X123 = A123(+A1232*LOG(K123)+A1233*LOG(L123)
758:      X124 = A1241+A1242*LOG(K124)+A1243*LOG(L124)
759:      X125 = A12(+A1252*LOG(K125)+A1253*LOG(L125)
760:      D121 = AA1211+AA1212*K121
761:      D122 = AA1221+AA1222*K122
762:      D123 = AA1231+AA1232*K123
763:      D124 = AA1241+AA1242*K124
764:      D125 = AA1251+AA1252*K125
765:      L121 = E1211+E1212*N12
766:      L122 = E1221+E1222*N12
767:      L123 = E1231+E1232*N12
768:      L124 = E1241+E1242*N12
769:      L125 = E1251+E1252*N12
770:      W121 = F1211+F1212*Y121
771:      W122 = F1221+F1222*Y122
772:      W123 = F1231+F1232*Y123
773:      W124 = F1241+F1242*Y124
774:      W125 = F1251+F1252*Y125
775:      T12 = G121+G122*(X121+X122+X123+X124+X125)
776:      U122 = H121+H122*(W121+W122+W123+W124+W125)
777:      B12 = J121+J122*B
778:      Q122 = V1221+V1222*T
779:      K131 = K131(-1)+B1311+B1312*I131+B1313*I131(-1)+B1314*I131(-2)+B1315*EX
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780:    K132 = K132(-1)+B1321+B1322*I132+B1323*I132(-1)+B1324*I132(-2)+B1325*EX
781:    K133 = K133(-1)+B1331+B1332*I133+B1333*I133(-1)+B1334*I133(-2)+B1335*EX
782:    K134 = K134(-1)+B1341+B1342*I134+B1343*I134(-1)+B1344*I134(-2)+B1345*EX
783:    K135 = K135(-1)+B1351+B1352*I135+B1353*I135(-1)+B1354*I135(-2)+B1355*EX
784:    DD131 = C1311+C1312*D131
785:    DD132 = C1321+C1322*D132
786:    DD133 = C1331+C1332*D133
787:    DD134 = C1341+C1342*D134
788:    DD135 = C1351+C1352*D135
789:    II131 = CC1311+CC1312*B13
790:    II132 = CC1321+CC1322*B13
791:    II133 = CC1331+CC1332*B13
792:    II134 = CC1341+CC1342*B13
793:    II135 = CC1351+CC1352*B13
794:    X131 = A1311+A1312*LOG(K131)+A1313*LOG(L131)
795:    X132 = A1321+A1322*LOG(K132)+A1323*LOG(L132)+A1324*LOG(Q132)
796:    X133 = A1331+A1332*LOG(K133)+A1333*LOG(L133)
797:    X134 = A1341+A1342*LOG(K134)+A1343*LOG(L134)
798:    X135 = A1351+A1352*LOG(K135)+A1353*LOG(L135)
799:    D131 = AA1311+AA1312*K131
800:    D132 = AA1321+AA1322*K132
801:    D133 = AA1331+AA1332*K133
802:    D134 = AA1341+AA1342*K134
803:    D135 = AA1351+AA1352*K135
804:    L131 = E1311+E1312*N13
805:    L132 = E1321+E1322*N13
806:    L133 = E1331+E1332*N13
807:    L134 = E1341+E1342*N13
808:    L135 = E1351+E1352*N13
809:    W131 = F1311+F1312*Y131
810:    W132 = F1321+F1322*Y132
811:    W133 = F1331+F1332*Y133
812:    W134 = F1341+F1342*Y134
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813:      W135 = F1351+F1352*Y135
814:      T13 = G131+G132*(X131+X132+X133+X134+X135)
815:      U132 = H131+H132*(W131+W132+W133+W134+W135)
816:      B13 = J131+J132*B
817:      Q132 = V1321+V1322*T
818:      K141 = K141(-1)+B1411+B1412*I141+B1413*I141(-1)+B1414*I141(-2)+B1415*EX
819:      K142 = K142(-1)+B1421+B1422*I142+B1423*I142(-1)+B1424*I142(-2)+B1425*EX
820:      K143 = K143(-1)+B1431+B1432*I143+B1433*I143(-1)+B1434*I143(-2)+B1435*EX
821:      K144 = K144(-1)+B1441+B1442*I144+B1443*I144(-1)+B1444*I144(-2)+B1445*EX
822:      K145 = K145(-1)+B1451+B1452*I145+B1453*I145(-1)+B1454*I145(-2)+B1455*EX
823:      DD141 = C1411+C1412*D141
824:      DD142 = C1421+C1422*D142
825:      DD143 = C1431+C1432*D143
826:      DD144 = C1441+C1442*D144
827:      DD145 = C1451+C1452*D145
828:      II141 = CC1411+CC1412*B14
829:      II142 = CC1421+CC1422*B14
830:      II143 = CC1431+CC1432*B14
831:      II144 = CC1441+CC1442*B14
832:      II145 = CC1451+CC1452*B14
833:      X141 = A1411+A1412*LOG(K141)+A1413*LOG(L141)
834:      X142 = A1421+A1422*LOG(K142)+A1423*LOG(L142)+A1424*LOG(Q142)
835:      X143 = A1431+A1432*LOG(K143)+A1433*LOG(L143)
836:      X144 = A1441+A1442*LOG(K144)+A1443*LOG(L144)
837:      X145 = A1451+A1452*LOG(K145)+A1453*LOG(L145)
838:      D141 = AA1411+AA1412*K141
839:      D142 = AA1421+AA1422*K142
840:      D143 = AA1431+AA1432*K143
841:      D144 = AA1441+AA1442*K144
842:      D145 = AA1451+AA1452*K145
843:      L141 = E1411+E1412*N14
844:      L142 = E1421+E1422*N14
845:      L143 = E1431+E1432*N14
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846:      L144 = E1441+E1442*N14
847:      L145 = E1451+E1452*N14
848:      W141 = F1411+F1412*Y141
849:      W142 = F1421+F1422*Y142
850:      W143 = F1431+F1432*Y143
851:      W144 = F1441+F1442*Y144
852:      W145 = F1451+F1452*Y145
853:      T14 = G141+G142*(X141+X142+X143+X144+X145)
854:      U142 = H141+H142*(W141+W142+W143+W144+W145)
855:      B14 = J141+J142*B
856:      Q142 = V1421+V1422*T
857:      K151 = K151(-1)+B1511+B1512*I151+B1513*I151(-1)+B1514*I151(-2)+B1515*EX
858:      K152 = K152(-1)+B1521+B1522*I152+B1523*I152(-1)+B1524*I152(-2)+B1525*EX
859:      K153 = K153(-1)+B1531+B1532*I153+B1533*I153(-1)+B1534*I153(-2)+B1535*EX
860       K154 = K154(-1)+B1541+B1542*I154+B1543*I154(-1)+B1544*I154(-2)+B1545*EX
861:      K155 = K155(-1)+B1551+B1552*I155+B1553*I155(-1)+B1554*I155(-2)+B1555*EX
862:      DD151 = C1511+C1512*D151
863:      DD152 = C1521+C1522*D152
864:      DD153 = C1531+C1532*D153
865:      DD154 = C1541+C1542*D154
866:      DD155 = C1551+C1552*D155
867:      II151 = CC1511+CC1512*B15
868:      II152 = CC1521+CC1522*B15
869:      II153 = CC1531+CC1532*B15
870:      II154 = CC1541+CC1542*B15
871:      II155 = CC1551+CC1552*B15
872:      X151 = A1511+A1512*LOG(K151)+A1513*LOG(L151)
873:      X152 = A1521+A1522*LOG(K152)+A1523*LOG(L152)+A1524*LOG(Q152)
874:      X153 = A1531+A1532*LOG(K153)+A1533*LOG(L153)
875:      X154 = A1541+A1542*LOG(K154)+A1543*LOG(L154)
876:      X155 = A1551+A1552*LOG(K155)+A1553*LOG(L155)
877:      D151 = AA1511+AA1512*K151
878:      D152 = AA1521+AA1522*K152
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879:        D153 = AA1531+AA1532*K153
880:        D154 = AA1541+AA1542*K154
881:        D155 = AA1551+AA1552*K155
882:        L151 = E1511+E1512*N15
883:        L152 = E1521+E1522*N15
884:        L153 = E1531+E1532*N15
885:        L154 = E1541+E1542*N15
886:        L155 = E1551+E1552*N15
887:        W151 = F1511+F1512*Y151
888:        W152 = F1521+F1522*Y152
889:        W153 = F1531+F1532*Y153
890:        W154 = F1541+F1542*Y154
891:        W155 = F1551+F1552*Y155
892:        T15 = G151+G152*(X151+X152+X153+X154+X155)
893:        U152 = H151+H152*(W151+W152+W153+W154+W155)
894:        B15 = J151+J152*B
895:        Q152 = V1521+V1522*T
```